



**Zagazig University
Faculty of Science
Physics Department**



General Physics

Lecture Notes

**For
First Year Students**

**By
Physics Department**

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General PHYSICS

Introduction

*The goal of physics is to provide an understanding of the physical world by developing theories based on experiments. A physical theory is essentially a guess, usually expressed mathematically, about how a given physical system works. The theory makes certain predictions about the physical system which can then be checked by observations and experiments. If the predictions turn out to correspond closely to what is actually observed, then the theory stands, although it remains provisional. No theory to date has given a complete description of *all physical phenomena*, even within a given sub-discipline of physics. Every theory is a work in progress. The feature study in this term involves " ***Electricity and magnetism – Geometrical optics – Properties of matter and heat*** "*

The basic laws of physics involve such physical quantities as force, velocity, volume, and acceleration, all of which can be described in terms of more fundamental quantities. The three most fundamental quantities are length (L), mass(M), and time (T); all other physical quantities can be constructed from these three.

DIMENSIONAL ANALYSIS

*In physics, it's often necessary either to derive a mathematical expression or equation or to check its correctness. A useful procedure for doing this is called dimensional analysis, which makes use of the fact that dimensions can be treated as algebraic quantities. The quantity x has the **dimension length**: $x = L$. Time t , of course has dimension $t = T$. **Acceleration** is the change in velocity v with time. Since v has dimensions of length per unit time, or $v = L/T$, **acceleration** must have dimensions $a = L/T^2$. We organize this information in the form of an equation:*

$$[\text{Acceleration } a] = \frac{[v]}{[t]} = \frac{L/T}{T} = \frac{L}{T^2} = \frac{[x]}{[t]^2}$$

$$[\text{Force } F] = [ma] = \left[\frac{mv}{t} \right] = \left[\frac{mx}{t^2} \right]$$

Table(1)

Dimensions and Some units of Area, Volume				
System	Area (L²)	Volume (L³)	Velocity (L/T)	Acceleration (L/T²)
SI	m^2	m^3	m/s	m/s^2
cgs	cm^2	cm^3	cm/s	cm/s^2
U.S	ft^2	ft^3	ft/s	ft/s^2

Chapter **I**

GEOMETRIC OPTICS

Chapter I " Geometric Optics"

We study the fundamental phenomena of geometric optics—reflection of light from a surface and refraction as the light crosses the boundary between two media. We will also study the dispersion of light as it refracts into materials, resulting in visual displays such as the rainbow. Finally, we investigate the phenomenon of total internal reflection, which is the basis for the operation of optical fibers and the burgeoning technology of fiber optics.

1. The Ray Approximation in Geometric Optics :

The field of geometric optics involves the study of the propagation of light, with the assumption that light travels in a fixed direction in a straight line as it passes through a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are non-uniform in either space or time. We use what is called *the ray approximation* for studying geometric optics . To understand this approximation, first note that *the rays of a given wave are straight lines perpendicular to the wave fronts* as illustrated in Figure 2 for a plane wave. **In the ray approximation**, we assume that *a wave moving through a medium travels in a straight line in the direction of its rays*.

2. Reflection

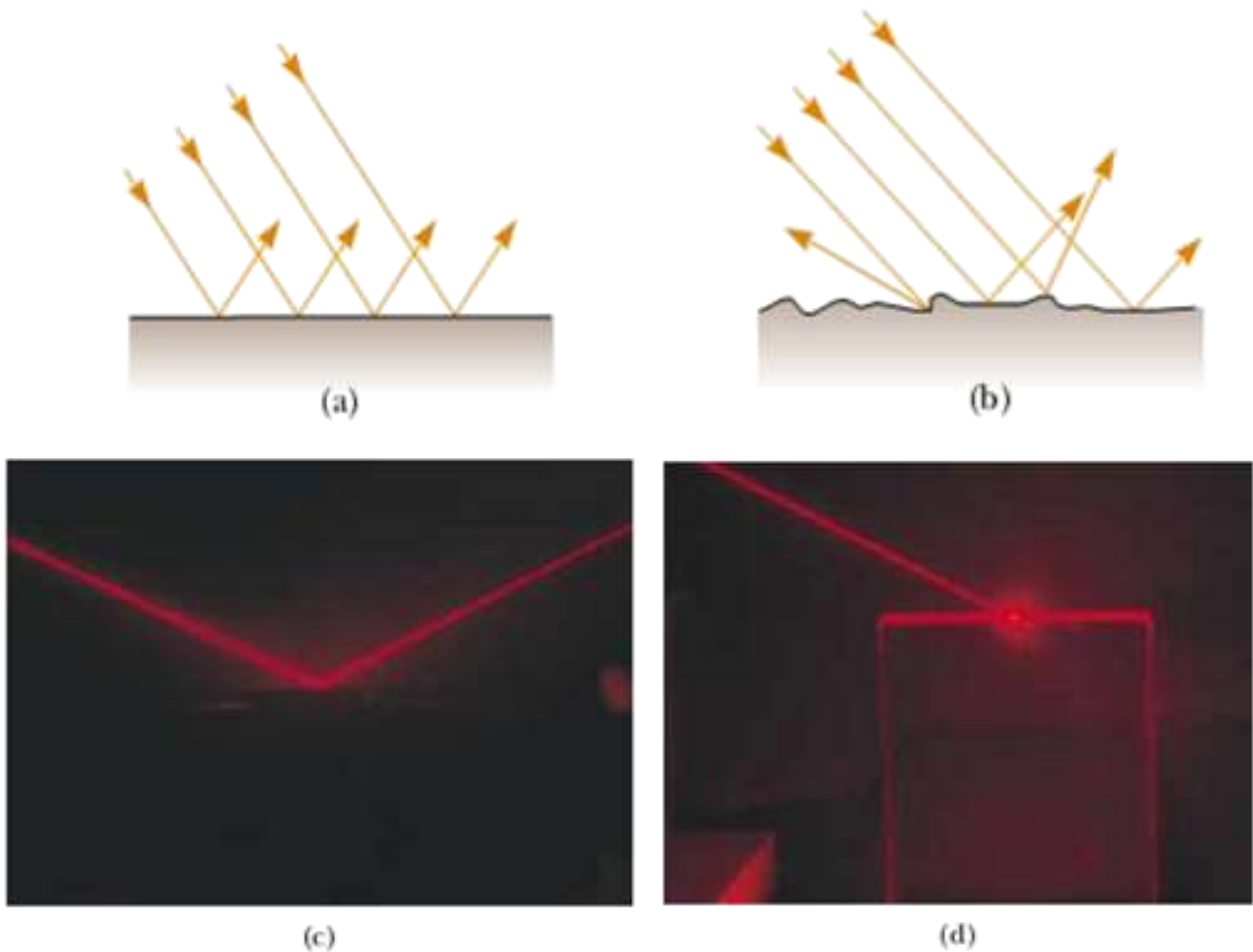


Fig.(1) Schematic representation of (a) specular reflection, where the reflected rays are all parallel to each other, and (b) diffuse reflection, where the reflected rays travel in random directions. (c) and (d) Photographs of specular and diffuse reflection using laser light.

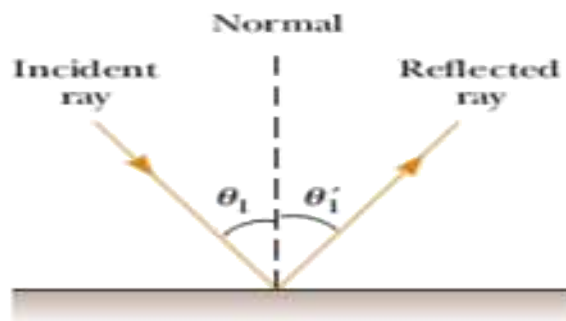


Fig.(2) According to the law of reflection $\theta_i = \theta'_i$. The incident ray, the reflected ray, and the normal all lie in the same plane.

When a light ray traveling in one medium encounters a boundary with another medium, part of the incident light is reflected. Figure 1a shows several rays of a beam of light incident on a smooth, mirror-like, reflecting surface. The reflected rays are parallel to each other, as indicated in the figure. The direction of a reflected ray is in the plane perpendicular to the reflecting surface that contains the incident ray. Reflection of light from such a smooth surface is called specular Reflection . If the reflecting surface is rough, as shown in Figure 1b, the surface reflects the rays not as a parallel set but in various directions. Reflection from any rough surface is known as diffuse reflection. A surface behaves as a smooth surface as long as the surface variations are much smaller than the wavelength of the incident light.

In geometric optics, we use the ray approximation, in which a wave travels through a uniform medium in straight lines in the direction of the rays. *The law of reflection states that for a light ray traveling in air and incident on a smooth surface, the angle of reflection θ'_1 equals the angle of incidence θ_1 as shown in Fig.(2) :*

$$\theta_1 = \theta'_1. \quad (1)$$

3. Refraction

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, as shown in Figure 3, part of the energy is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be refracted. The incident ray, the reflected ray, and the refracted ray all lie in the same plane.

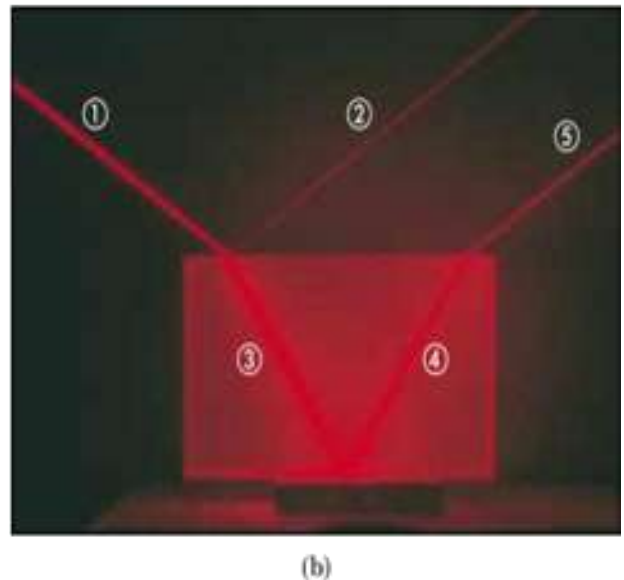
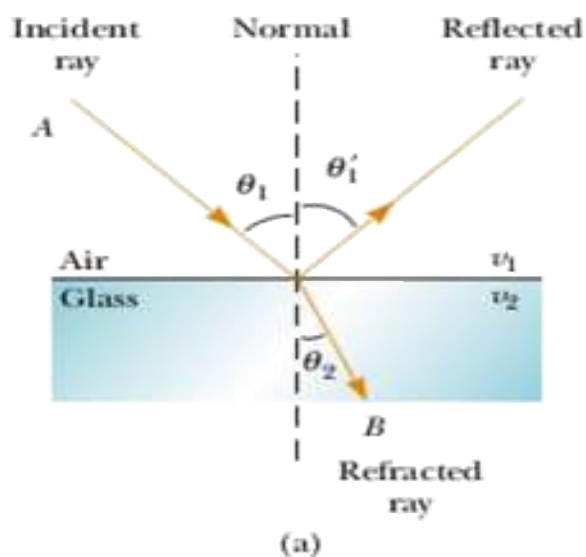


Fig.(3) (a) A ray obliquely incident on an air–glass interface. The refracted ray is bent toward the normal because $v_2 < v_1$. All rays and the normal lie in the same plane. (b) Light incident on the Lucite block bends both when it enters the block and when it leaves the block.

The angle of refraction θ_2 in Figure 3a, depends on the properties of the two media and on the angle of incidence through the relationship

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant} \quad (2)$$

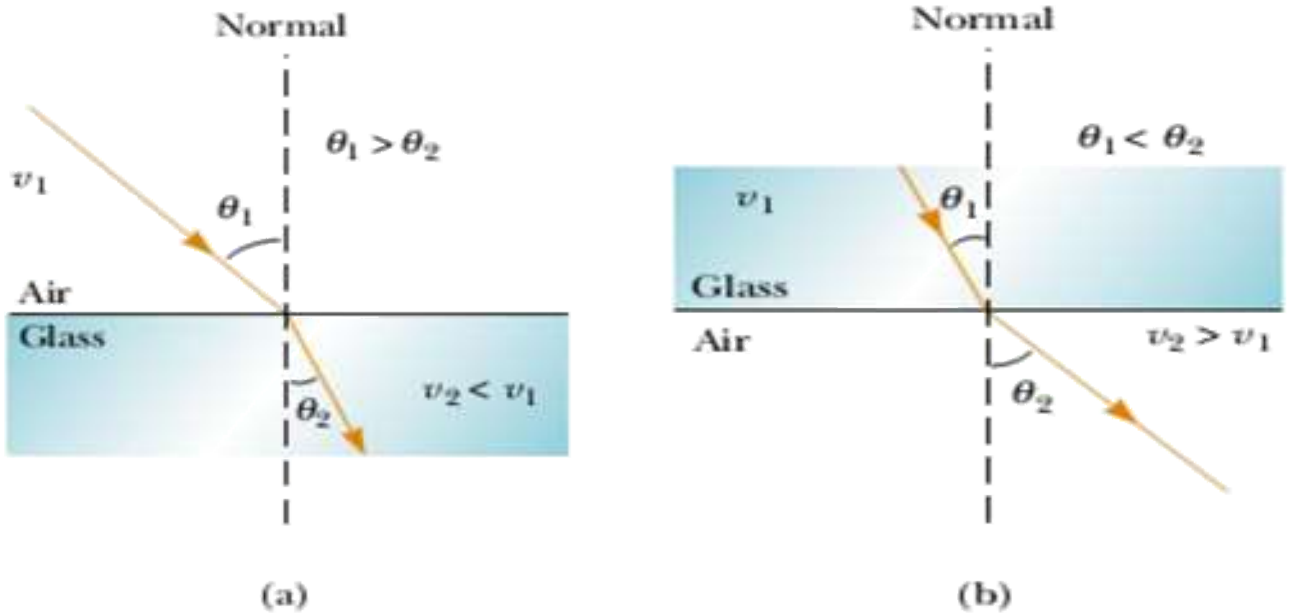


Fig.(4) (a) When the light beam moves from air into glass, the light slows down on entering the glass and its path is bent toward the normal. (b) When the beam moves from glass into air, the light speeds up on entering the air and its path is bent away from the normal.

4. Index of Refraction

In general, the speed of light in any material is less than its speed in vacuum. In fact, *light travels at its maximum speed in vacuum.*

It is convenient to define the ***index of refraction*** of a medium to be the ratio

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \quad (3)$$

where c is the speed of light in a vacuum and v is the speed of light

in the medium. In general, n varies with wavelength as shown in Fig.(5) and is given by

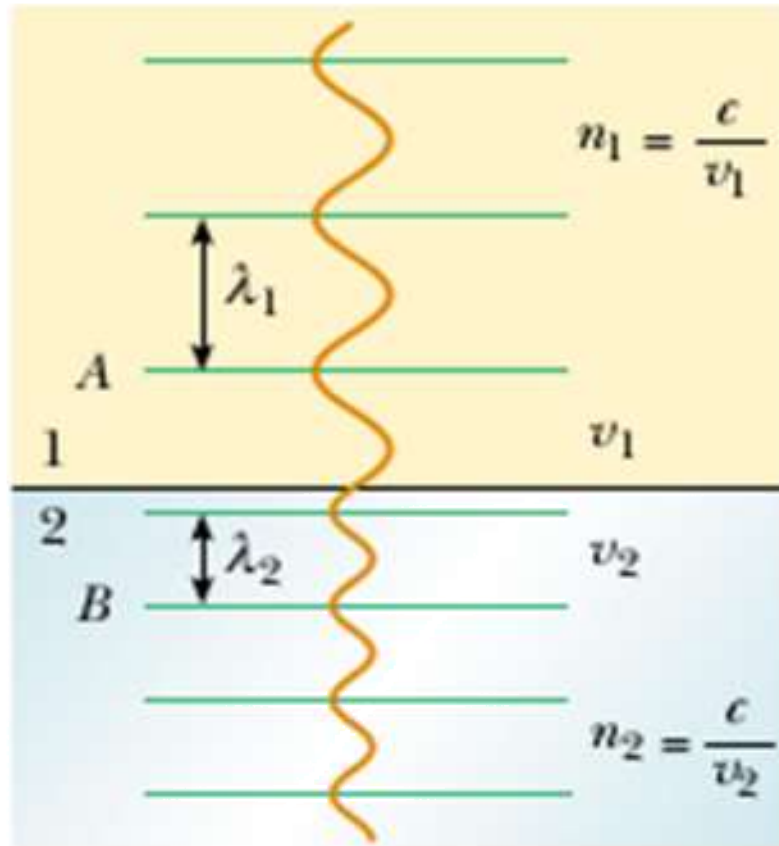


Fig.(5) As a wave moves from medium 1 to medium 2, its wavelength changes but its frequency remains constant.

$$n = \frac{\lambda}{\lambda_n} \quad (4)$$

where λ is the vacuum wavelength and λ_n is the wavelength in the medium. As light travels from one medium to another, its frequency remains the same. Snell's law of refraction states that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (5)$$

where n_1 and n_2 are the indices of refraction in the two media. The incident ray, the reflected ray, the refracted ray, and the normal to the surface all lie in the same plane.

5. Huygens's Principle Applied to Reflection and Refraction

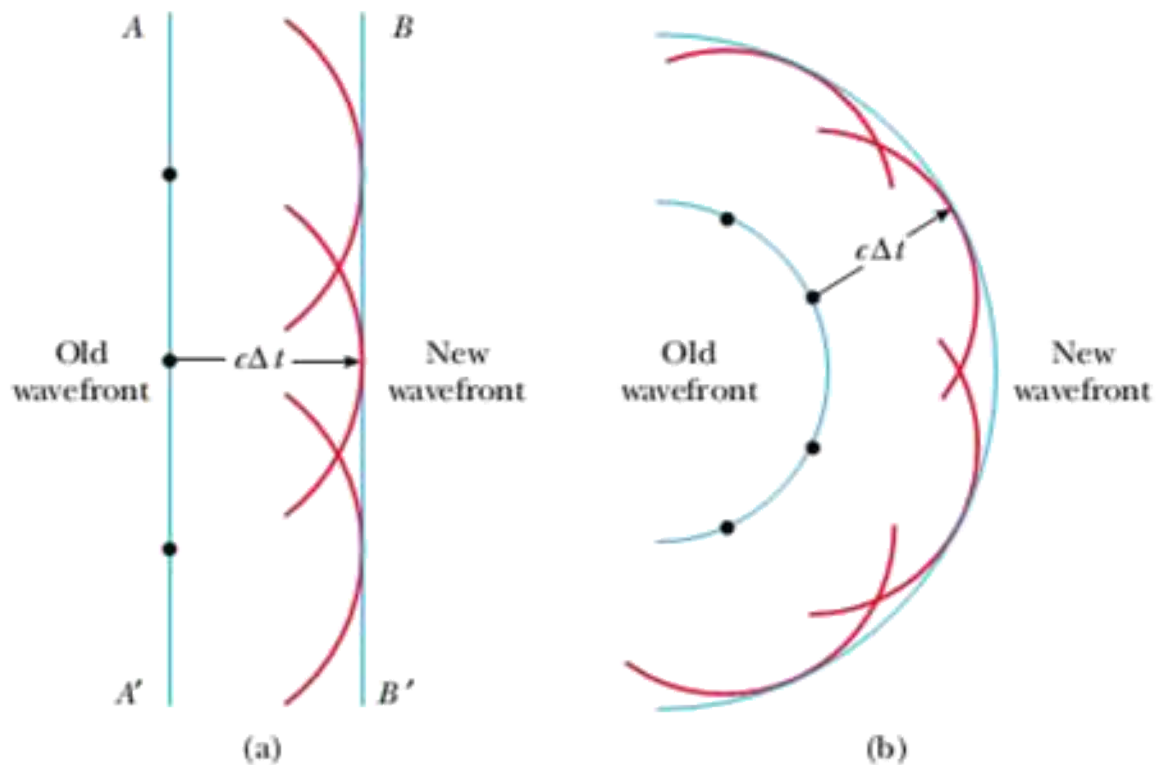


Fig.(6) Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

The laws of reflection and refraction can be derived by using Huygens's principle , figure 6 shows this principle as" *all points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outward through a medium with speeds characteristic of waves in*

that medium. After some time interval has passed, the new position of the wave front is the surface tangent to the wavelets."

We apply this principle for the law of reflection, shown in figures 6,7. The line AB represents a wave front of the incident light just as ray 1 strikes the surface. At this instant, the wave at A sends out a Huygens wavelet (the circular arc centered on A) toward D. At the same time, the wave at B emits a Huygens wavelet (the circular arc centered on B) toward C. Figure 6a shows these wavelets after a time interval Δt , after which ray 2 strikes the surface. Because both rays 1 and 2 move with the same speed, we must have $AD = BC = c \Delta t$. The remainder of our analysis depends on geometry, as summarized in Figure 6b, in which we isolate the triangles ABC and ADC. Note that these two triangles are congruent because they have the same hypotenuse AC and because $AD = BC$. From Figure 6b, we have

$$\cos \gamma = \frac{BC}{AC} \quad \text{and} \quad \cos \gamma' = \frac{AD}{AC} \quad (6)$$

Therefore,

$$\gamma = \gamma'$$

$$90^\circ - \theta_i = 90^\circ - \theta'_i$$

and

$$\theta_1 = \theta'_1$$

which is the law of reflection.

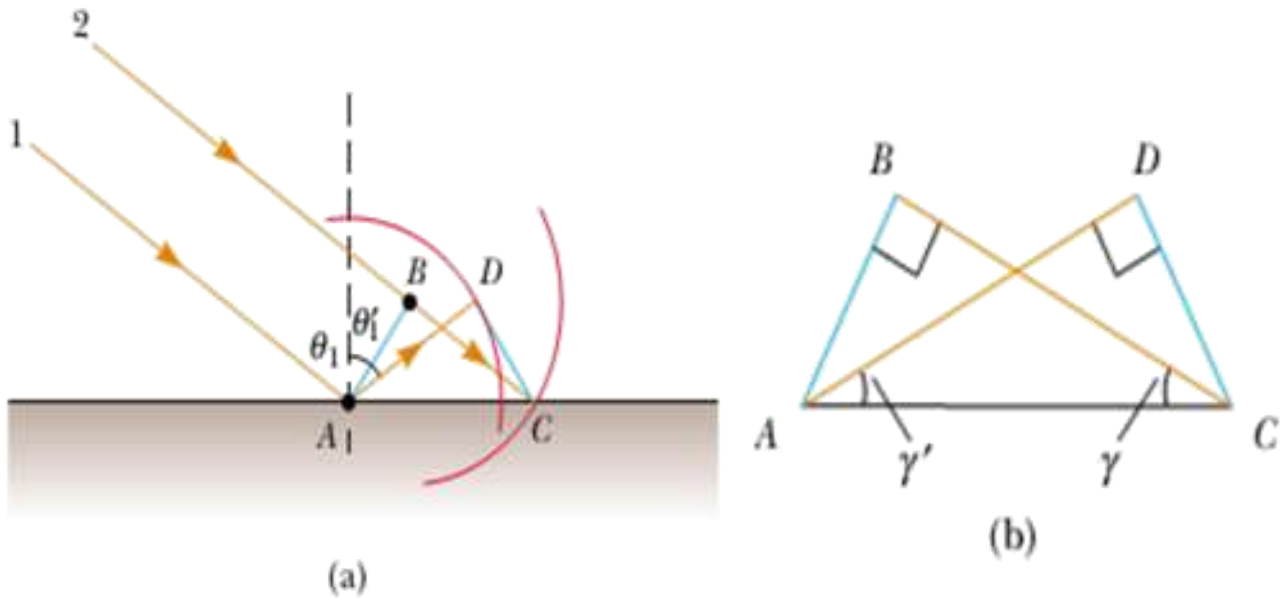


Fig.(6) (a) Huygens's construction for proving the law of reflection. At the instant that ray 1 strikes the surface, it sends out a Huygens wavelet from A and ray 2 sends out a Huygens wavelet from B. We choose a radius of the wavelet to be $c\Delta t$, where Δt is the time interval for ray 2 to travel from B to C. (b) Triangle ADC is congruent to triangle ABC.

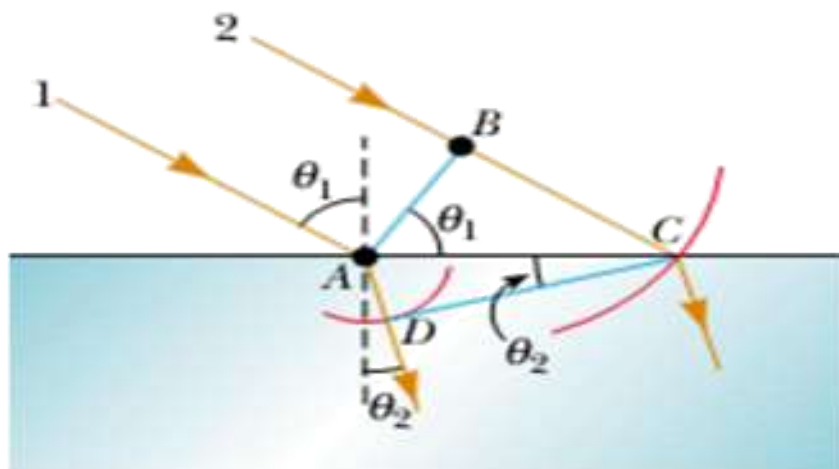


Fig.(7) Huygens's construction for proving Snell's law of refraction. At the instant that ray 1 strikes the surface, it sends out a Huygens wavelet from A

and ray 2 sends out a Huygens wavelet from B. The two wavelets have different radii because they travel in different media.

Snell's law of refraction can be derived by using Huygens's principle and Figure 7. the instant ray 1 strikes the surface and the subsequent time interval. During this time interval, the wave at A sends out a Huygens wavelet (the arc centered on A) toward D.

In the same time interval, the wave at B sends out a Huygens wavelet (the arc centered on B) toward C. Because these two wavelets travel through different media, the radii of the wavelets are different. The radius of the wavelet from A is $AD = v_2 \Delta t$, where v_2 is the wave speed in the second medium. The radius of the wavelet from B is $BC = v_1 \Delta t$, where v_1 is the wave speed in the original medium.

From triangles ABC and ADC, we find that

$$\sin \theta_1 = \frac{BC}{AC} = \frac{v_1 \Delta t}{AC} \quad \text{and} \quad \sin \theta_2 = \frac{AD}{AC} = \frac{v_2 \Delta t}{AC}$$

If we divide the first equation by the second, we obtain

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} \quad (7)$$

But we know that $v_1 = c/n_1$ and $v_2 = c/n_2$. Therefore

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \quad (8)$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (9)$$

which is Snell's law of refraction .

6. Dispersion through Prisms

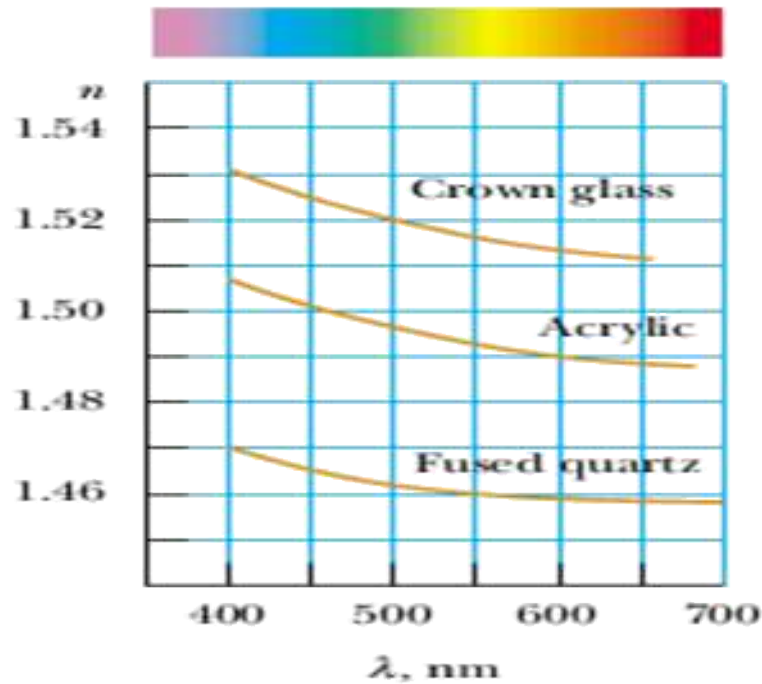


Fig.(8) Variation of index of refraction with vacuum wavelength for three materials.

From the optical studies the refractive index n of the materials plays important role in dispersion phenomenon. The refractive index n varies with the wavelength of the light passing through the material, as Figure 8 shows. This behavior is called **dispersion**. Because n is a function of wavelength, Snell's law of refraction indicates that light of different wavelengths is bent at different angles when incident on a refracting material. Figure 8 shows that

violet light bends more than red light does when passing into a refracting material. The effects that dispersion happen when light strikes a prism, as shown in Figure 9. A ray of single-wavelength light incident on the prism from the left emerges refracted from its original direction of travel by an angle δ , called **the angle of deviation**.

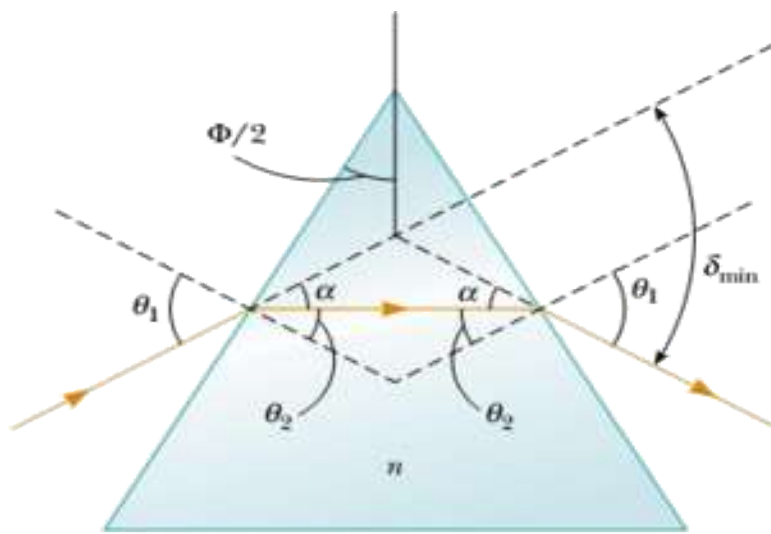


Fig.(11) A light ray passing through a prism at the minimum angle of deviation $\delta_{\min}$.

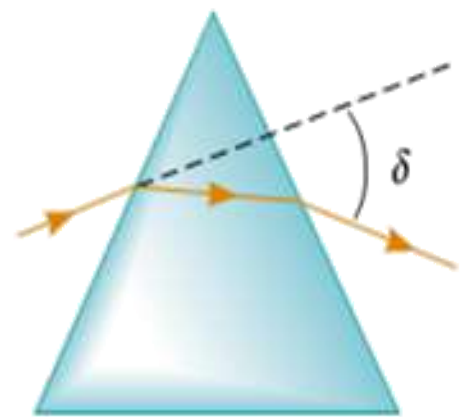


Fig.(10) A prism refracts a single-wavelength light ray through an angle δ .

The minimum angle of deviation $\delta_{\min}$ for a prism occurs when the angle of incidence θ_1 is such that the refracted ray inside the prism makes the same angle with the normal to the two prism faces, as shown in Figure 11. Using the geometry shown in Figure 11, we find that $\theta_2 = \Phi/2$, where Φ is the apex angle and

$$\theta_1 = \theta_2 + \alpha = \frac{\Phi}{2} + \frac{\delta_{\min}}{2} = \frac{\Phi + \delta_{\min}}{2}$$

From Snell's law of refraction, with $n_1 = 1$ because medium 1 is air, we have

$$\sin \theta_1 = n \sin \theta_2$$

$$n = \frac{\sin\left(\frac{\Phi + \delta_{\min}}{2}\right)}{\sin(\Phi/2)} \quad (10)$$

Hence, knowing the apex angle Φ of the prism and measuring $\delta_{\min}$, we can calculate the index of refraction of the prism material. Furthermore, we can use a hollow prism to determine the values of n for various liquids filling the prism.

7. Total Internal Reflection

Total internal reflection occurs when light travels from a medium of high index of refraction to one

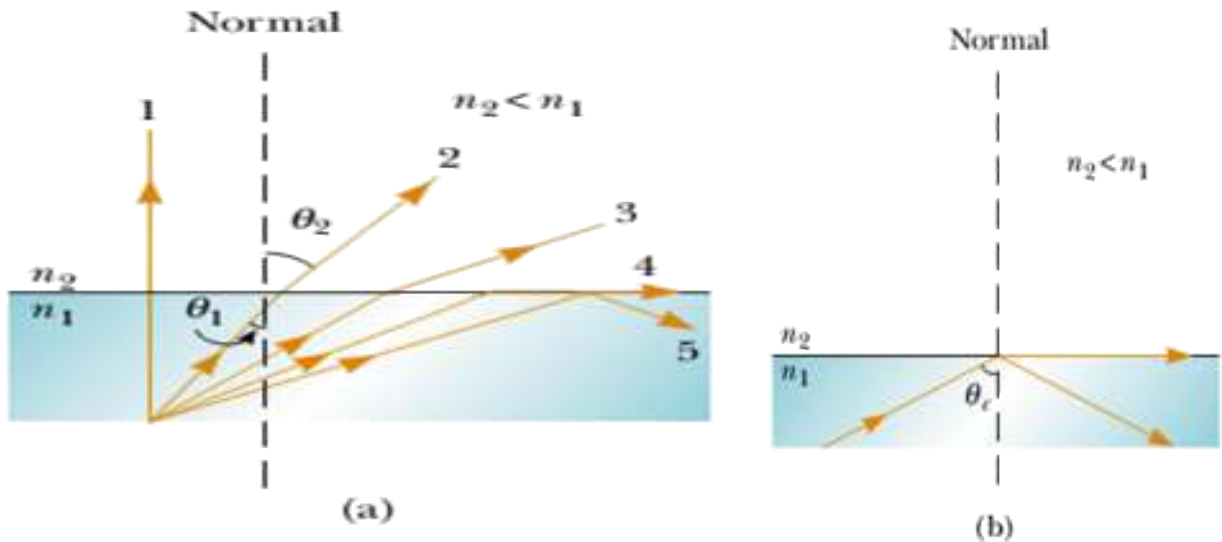


Fig.(12) (a) Rays travel from a medium of index of refraction n_1 into a medium of index of refraction n_2 , where $n_2 < n_1$. As the angle of incidence θ_1 increases, the angle of refraction θ_2 increases until θ_2 is 90° (ray 4). For even larger angles of incidence, total internal reflection

occurs (ray) .(5b)The angle of incidence producing an angle of refraction equal to 90° is the critical angle θ_c .At this angle of incidence, all of the energy of the incident light is reflected.
of lower index of refraction. The critical angle θ_c for which total internal reflection occurs at an interface is given by

$$\sin \theta_c = \frac{n_2}{n_1} \quad (\text{ for } n_1 > n_2) \quad (11)$$

8. Optical Fibers

It is one of the interesting applications of total internal refraction and plays important role in the optical devises . The optical fibers are constructed the glass or transparent plastic rods to “pipe” light from one place to another. The light is confined to traveling within a rod, even around curves, as the result of successive total internal reflections, as shown in figure 13. Such a light pipe is flexible if thin fibers are used rather than thick rods. A flexible light pipe is called an *optical fiber*. If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another. This technique is used in a sizable industry known as *fiber optics*.

A practical optical fiber consists of a transparent core surrounded by a cladding, a material that has a lower index of refraction than the core. The combination may be surrounded by a plastic jacket to prevent mechanical damage. Figure 14 shows a cutaway view of

this construction. Because the index of refraction of the cladding is less than that of the core, light traveling in the core experiences total internal reflection if it arrives at the interface between the core and the cladding at an angle of incidence that exceeds the critical angle. In this case, light “bounces” along the core of the optical fiber, losing very little of its intensity as it travels.

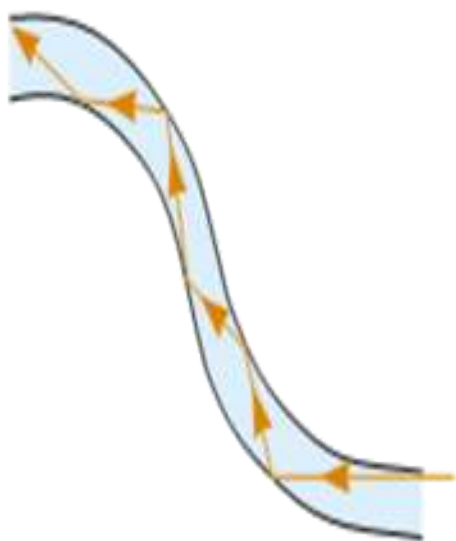


Fig.(13) Light travels in a curved transparent rod by multiple internal reflections.

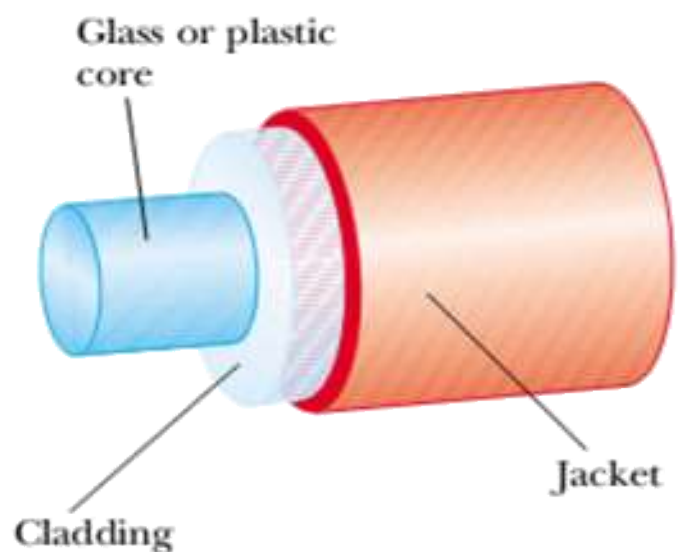


Fig.(14) The construction of an optical fiber. Light travels in the core, which is surrounded by a cladding and a protective jacket.

9. Fermat's Principle

*Fermat's Principle states that when a light ray travels between any two points, its path is the one that requires the smallest time interval . It can be used to derive **Snell's law of refraction**.* Suppose that a light ray is to travel from point P in medium 1 to point Q in medium 2 (Fig. 15), where P and Q are at perpendicular

distances a and b , respectively, from the interface. The speed of light is c/n_1 in medium 1 and c/n_2 in medium 2 . Using the geometry of Figure 15, and assuming that light leaves P at $t = 0$, we see that the time at which the ray arrives at Q is

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d - x)^2}}{c/n_2}$$

To obtain the value of x for which t has its minimum value, we take the derivative of t with respect to x and set the derivative equal to zero:

$$\frac{dt}{dx} = \frac{n_1}{c} \frac{d}{dx} \sqrt{a^2 + x^2} + \frac{n_2}{c} \frac{d}{dx} \sqrt{b^2 + (d - x)^2} = 0$$

or
$$\frac{n_1 x}{(a^2 + x^2)^{1/2}} = \frac{n_2 (d - x)}{[b^2 + (d - x)^2]^{1/2}} \quad (*)$$

From Figure 15,

$$\sin \theta_1 = \frac{x}{(a^2 + x^2)^{1/2}} \quad \sin \theta_2 = \frac{d - x}{[b^2 + (d - x)^2]^{1/2}}$$

Substituting these expressions into Equation (*), we find that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (12)$$

which is Snell's law of refraction.

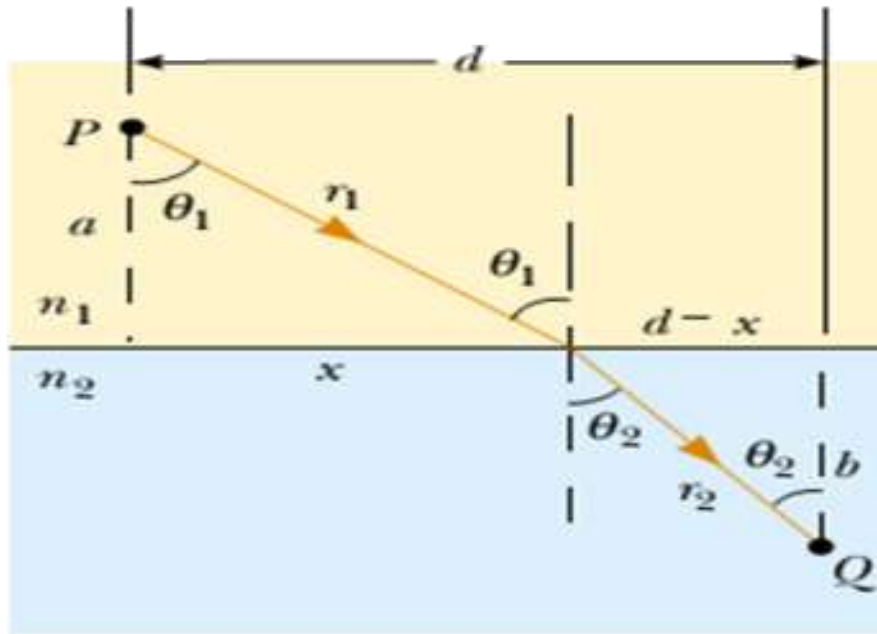


Fig.(15) Geometry for deriving Snell's law of refraction using Fermat's principle.

10. Image Formation by Mirrors

1. Images Formed by Flat Mirrors

Images are classified as *real* or *virtual*. A *real image* is formed when light rays pass through and diverge from the image point; a *virtual image* is formed when the light rays do not pass through the image point but only appear to diverge from that point. The image formed by the mirror in Figure 16 is virtual. The image of an object seen in a flat mirror is always virtual. Real images can be displayed on a screen (as at a movie), but virtual images cannot be displayed

on a screen. By using the simple geometry in Figure 17 we can examine the properties of the images of extended objects formed by flat mirrors.

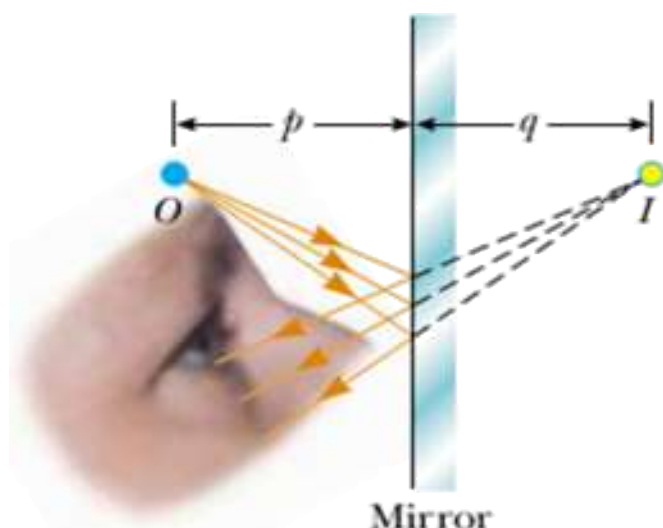


Fig.(16) An image formed by reflection from a flat mirror. The image point I is located behind the mirror a perpendicular distance q from the mirror (the image distance). The image distance has the same magnitude as the object distance p .

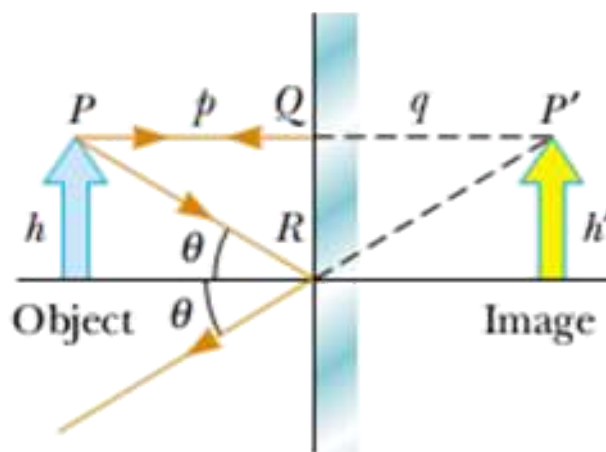


Fig.(17) A geometric construction that is used to locate the image of an object placed in front of a flat mirror. Because the triangles PQR and $P'QR$ are congruent, $|p| = |q|$ and $h = h'$.

An observer in front of the mirror would trace the two reflected rays back to the point at which they appear to have originated, which is point P' behind the mirror. A continuation of this process for points other than P on the object would result in a virtual image behind the mirror. Because triangles PQR and $P'QR$ are congruent, $PQ = P'Q$. We conclude that the image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror. Geometry also reveals that the

object height h equals the image height h' . Let us general define *lateral magnification* M of an image as follows :

$$M \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h} \quad (13)$$

Note : for a flat mirror, $M = 1$ for any image because $h' = h$.

2. Images Formed by Spherical Mirrors

2.a Concave Mirrors:

Such these mirrors, in which light is reflected from the inner, silvered concave surface, is called a concave mirror. Each mirror has a radius of curvature R , and its center of curvature is point C . Point V is the center of the spherical section, and a line through C and V is called the principal axis of the mirror.

We assume that O is a point source of light lies on the principal axis to the left of C , as shown in Figure . Two diverging rays that originate at O are shown. After reflecting from the mirror, these rays converge and cross at the image point I . They then continue to diverge from I as if an object were there. As a result, at point I we have a real image of the light source at O .

We can calculate the image distance q and height h' from a knowledge of the object height h and distance p , and radius of curvature R by using the geometric paths and mathematical

relations. The negative sign is introduced because the image is inverted, so h' is taken to be negative. Thus, from Equation 13 and these results, we find that the magnification of the image is

$$M = \frac{h'}{h} = -\frac{q}{p} \quad (14)$$

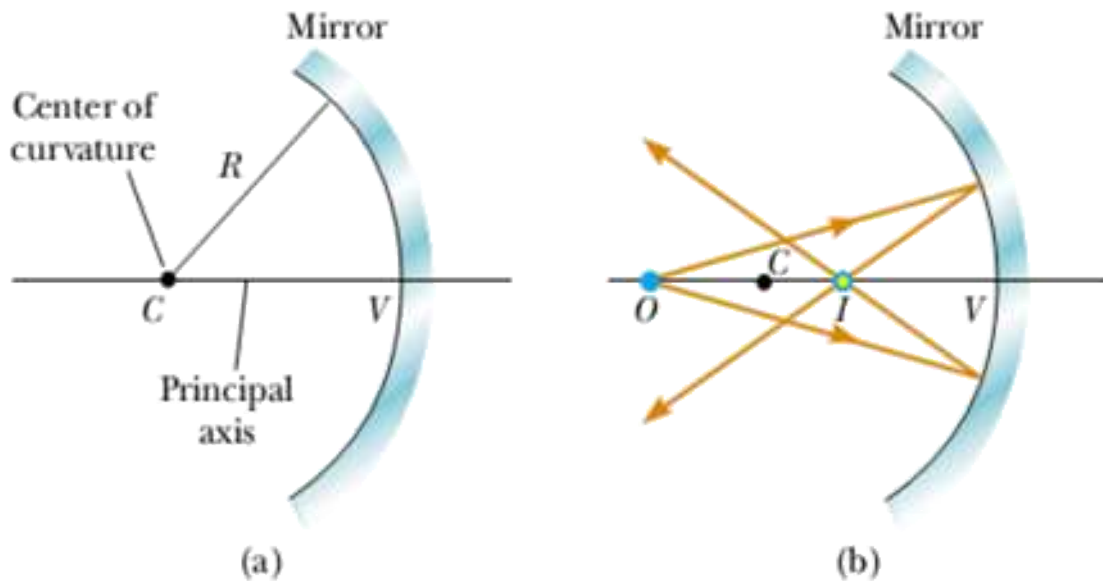


Fig.(18) (a) A concave mirror of radius R . The center of curvature C is located on the principal axis. (b) A point object placed at O in front of a concave spherical mirror of radius R , where O is any point on the principal axis farther than R from the mirror surface, forms a real image at I . If the rays diverge from O at small angles, they all reflect through the same image point.

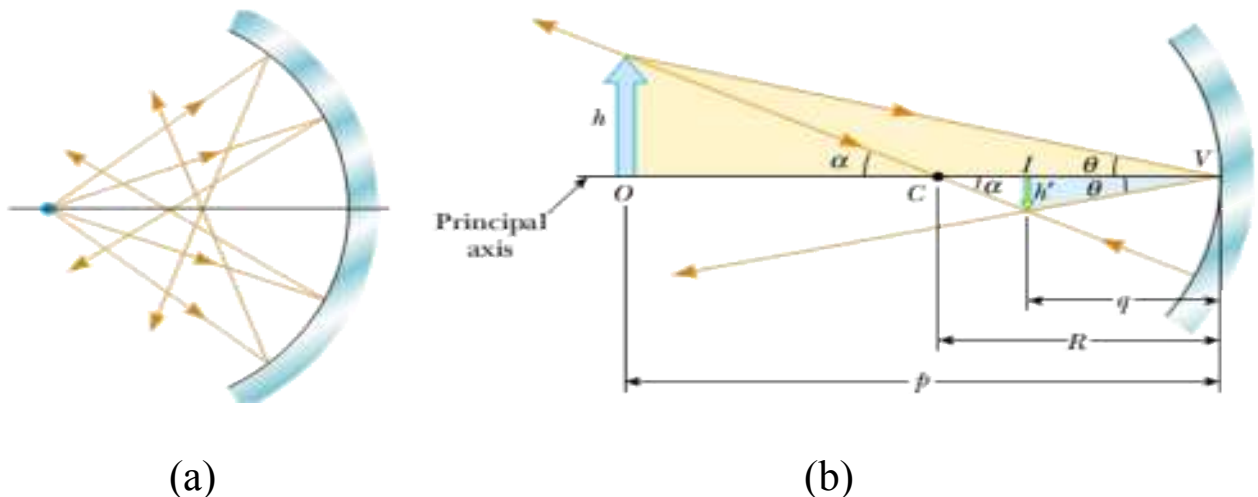


Fig.(19) (a) Rays diverging from the object at large angles from the principal axis reflect from a spherical concave mirror to intersect the principal axis at different points, resulting in a blurred image. This condition is called **spherical aberration**. (b) The image formed by a spherical concave mirror when the object **O** lies outside the center of curvature **C**. This geometric construction is used to derive Eq. 36.4

We also note from the two triangles in Figure 36.11 that have α as one angle that

$$\tan \alpha = \frac{h}{p-R} \quad \text{and} \quad \tan \alpha = -\frac{h'}{R-q}$$

from which we find that

$$\frac{h'}{h} = -\frac{R-q}{p-R}$$

Simple algebra reduces this to

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \quad (15)$$

This expression is called **the mirror equation**.

Assuming that the source is so far from the mirror and the incoming rays from the object are essentially parallel we should obtain the image point at focal point **F**, and the image distance is corresponded to the **focal length f**, where

$$f = \frac{R}{2}$$

The mirror equation can be expressed in terms of the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (16)$$

b) Convex Mirrors

The formation of an image by a convex mirror (or a diverging mirror) can be constructed, as shown in Figure 20, so that light is reflected from the outer, convex surface .

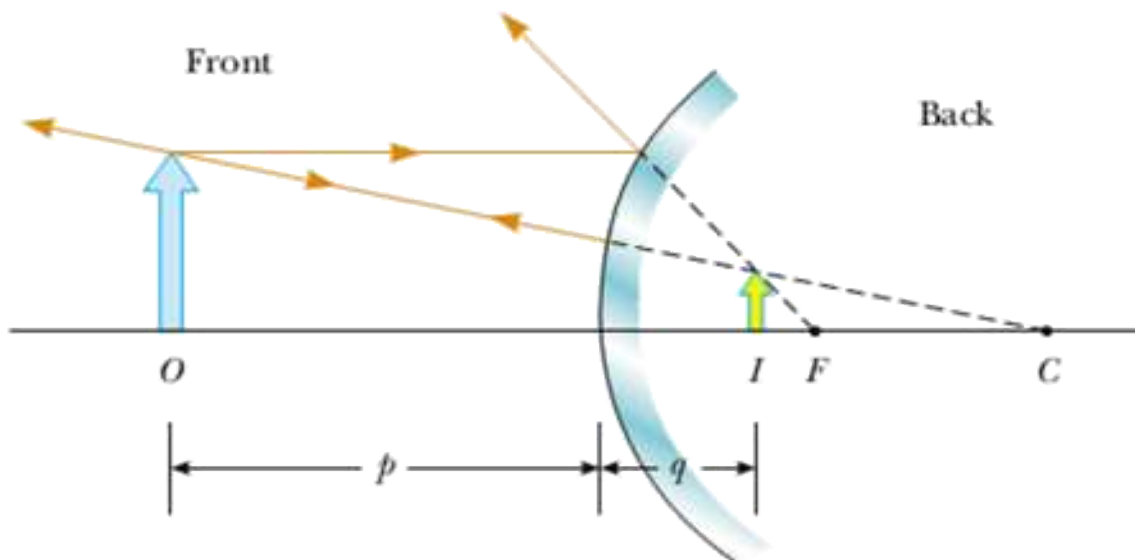
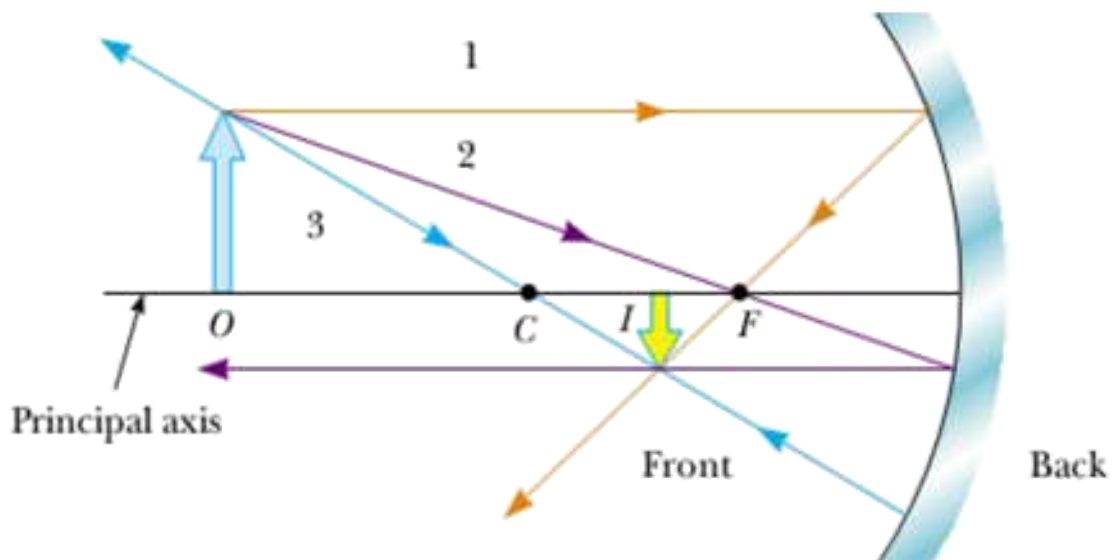


Fig.(20) Formation of an image by a spherical convex mirror. The image formed by the real object is virtual and upright.

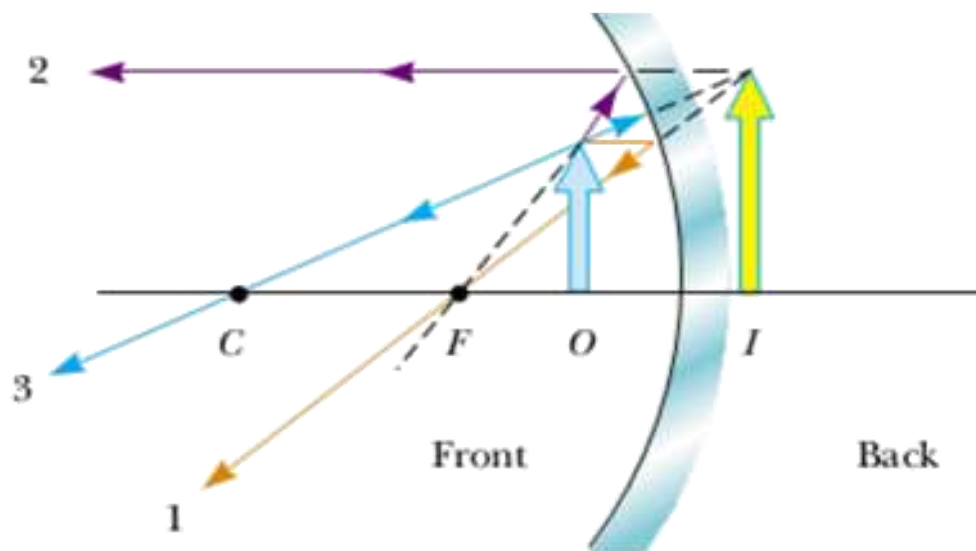
11. Ray Diagrams for Mirrors

The positions and sizes of images formed by mirrors can be conveniently determined with ray diagrams. These graphical constructions reveal the nature of the image and can be used to check results calculated from the mirror and magnification

equations. To draw a ray diagram, we need to know the *position* of the object and the *locations* of the mirror's focal point and *center of curvature*. We then draw three principal rays to locate the image, as shown by the examples in Figure 21.



(a)



(b)



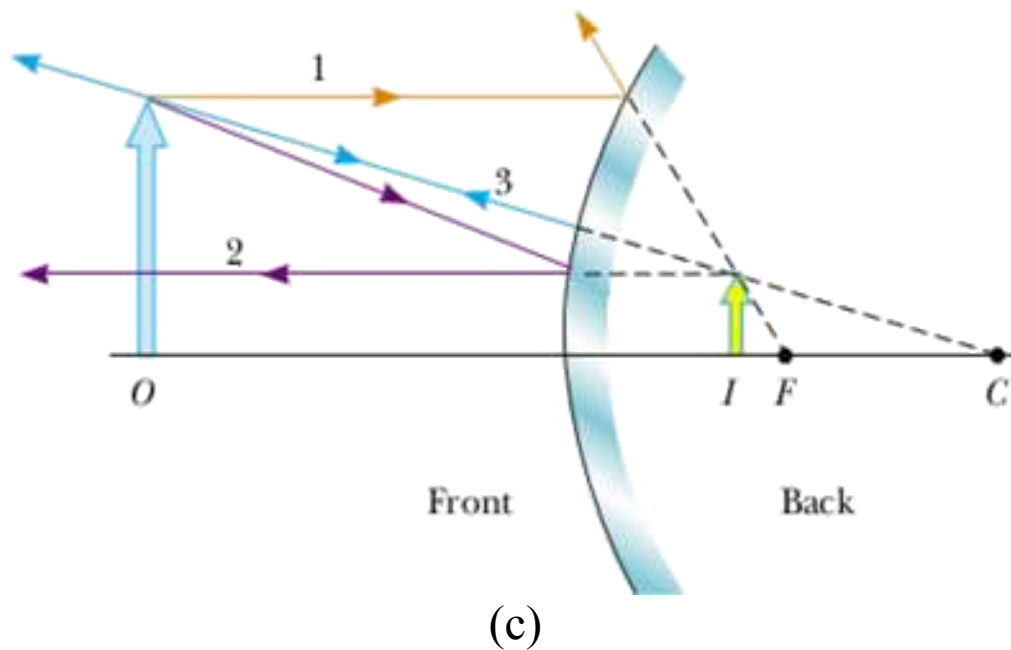


Fig.(21) Ray diagrams for spherical mirrors, along with corresponding photographs of the images of candles. (a) When the object is located so that the center of curvature lies between the object and a concave mirror surface, the image is real, inverted, and reduced in size. (b) When the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged. (c) When the object is in front of a convex mirror, the image is virtual, upright, and reduced in size.

These rays, three rays 1-2-3, all start from the same object point and are drawn as follows. We may choose any point on the object; here, we choose the top of the object for simplicity. *For concave mirrors (see Figs. 21a ,b), we draw the following three principal ray :*

- **Ray 1** is drawn from the top of the object parallel to the principal axis and is reflected through the focal point F .
- **Ray 2** is drawn from the top of the object through the focal point and is reflected parallel to the principal axis.

- **Ray 3** is drawn from the top of the object through the center of curvature C and is reflected back on itself.

For convex mirrors (see Fig. 21c), we draw the following three principal rays:

- **Ray 1** is drawn from the top of the object parallel to the principal axis and is reflected away from the focal point F .
- **Ray 2** is drawn from the top of the object toward the focal point on the back side of the mirror and is reflected parallel to the principal axis.
- **Ray 3** is drawn from the top of the object toward the center of curvature C on the back side of the mirror and is reflected back on itself.

12. Images Formed by Refraction

We assume that the object at O is in the medium for which the index of refraction is n_1 . Let us consider the paraxial rays leaving O . As we shall see, all such rays are refracted at the spherical surface and focus at a single point I , the image point. Figure 22 shows a single ray leaving point O and refracting to point I .

Snell's law of refraction applied to this ray gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (17)$$

Because θ_1 and θ_2 are assumed to be small, we can use the small-angle approximation $\sin \theta \approx \theta$ (with angles in radians) and say that

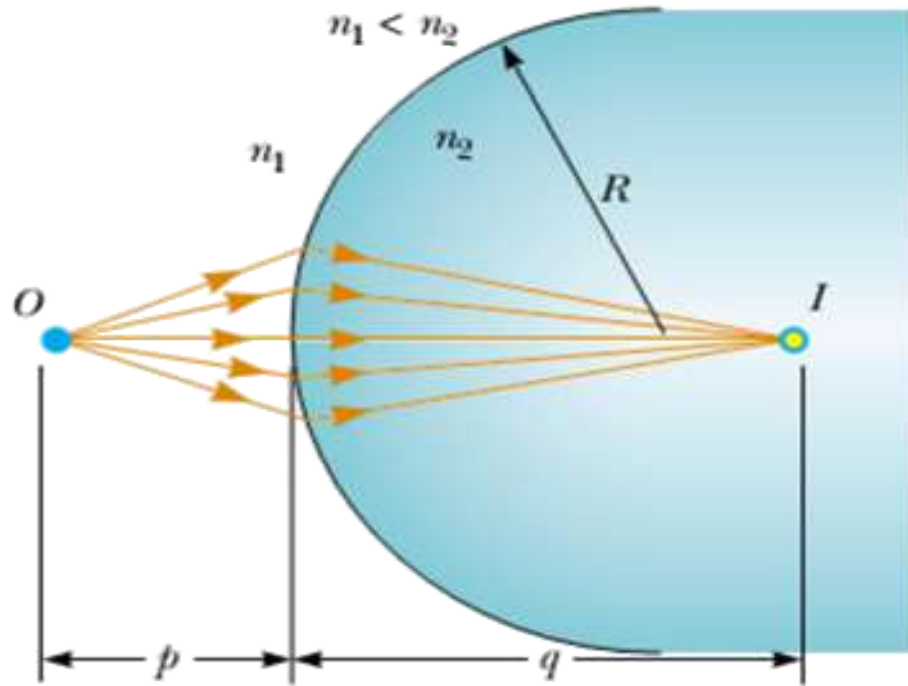


Fig.22 An image formed by refraction at a spherical surface. Rays making small angles with the principal axis diverge from a point object at O and are refracted through the image point I .

$$n_1 \theta_1 = n_2 \theta_2 \quad (18)$$

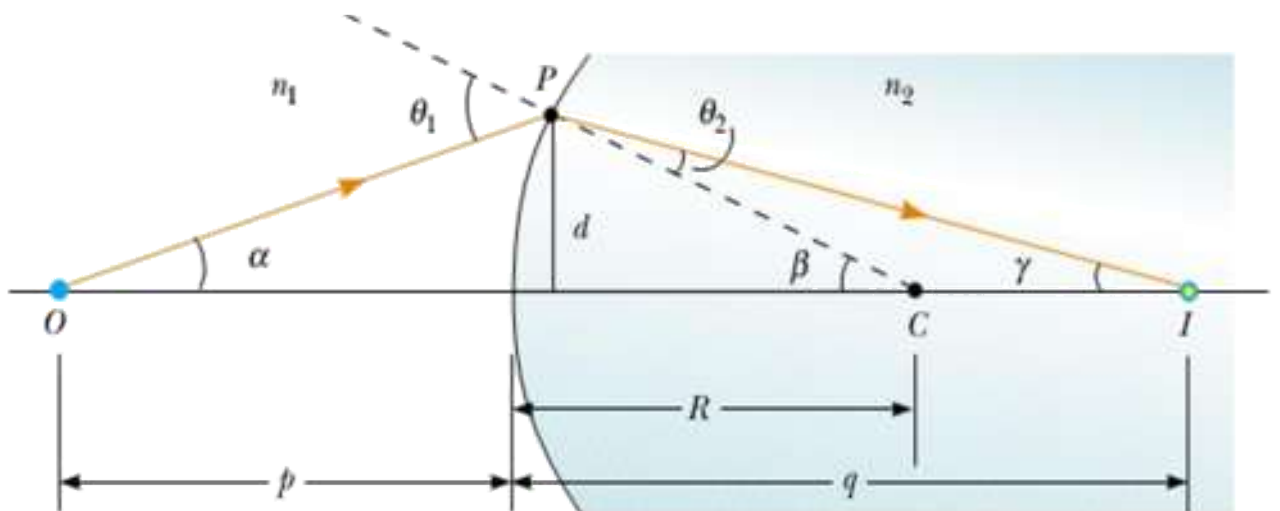


Fig.(23) Geometry used to derive Equation 36.8, assuming that $n_1 < n_2$.

Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this rule to triangles OPC and PIC in Figure 23 give

$$\theta_1 = \alpha + \beta , \quad \beta = \theta_2 + \gamma$$

If we combine all three expressions and eliminate θ_1 and θ_2 , we find that

$$n_1 \alpha + n_2 \gamma = (n_2 - n_1) \beta \quad (19)$$

From Figure 23, we see three right triangles that have a common vertical leg of length d . For paraxial rays (unlike the relatively large-angle ray shown in Fig. 23, (the horizontal legs of these triangles are approximately p for the triangle containing angle α , R for the triangle containing angle β , and q for the triangle containing angle γ . In the small-angle approximation, $\tan \theta \approx \theta$, so we can write the approximate relationships from these triangles as follows

$$\tan \alpha \approx \alpha \approx \frac{d}{p} , \quad \tan \beta \approx \beta \approx \frac{d}{R} , \quad \tan \gamma \approx \gamma \approx \frac{d}{q}$$

We substitute these expressions into Equation 19 and divide through by d to give

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \quad (20)$$

For a fixed object distance p , the image distance q is independent of the angle that the ray makes with the axis. This result tells us that all paraxial rays focus at the same point I . Equation 20 is valid regardless of which index of refraction is greater.

13. Images Formation by Thin Lenses

We recognize that light passing through a lens experiences refraction at two surfaces. The development we shall follow is based on the notion that the image formed by one refracting surface serves as the object for the second surface.

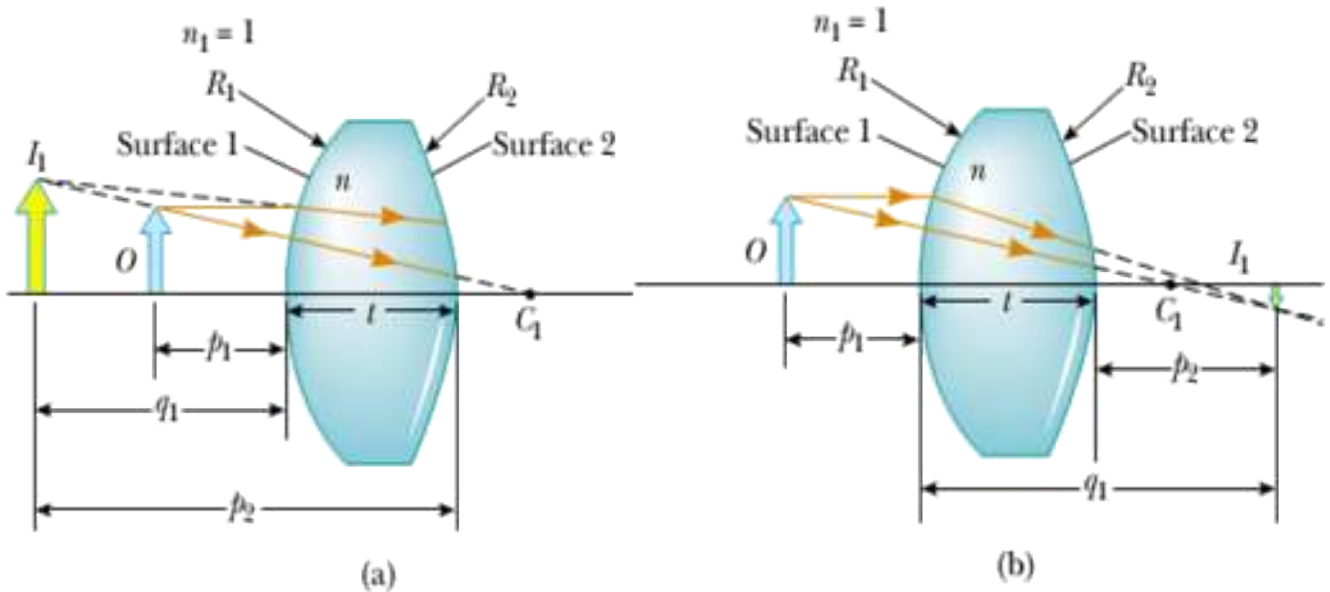


Fig.(24) To locate the image formed by a lens, we use the virtual image at I_1 formed by surface 1 as the object for the image formed by surface 2. The point C_1 is the center of curvature of surface 1. (a) The image due to surface 1 is virtual so that I_1 is to the left of the surface. (b) The image due to surface 1 is real so that I_1 is to the right of the surface.

We shall firstly analyze a **thick lens** and then let the thickness of the lens be approximately zero. Let us begin with the image formed by surface 1. Using Equation 20 and assuming that $n_1 = 1$ because

the lens is surrounded by air, we find that the image I_1 formed by surface 1 satisfies the equation

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1} \quad (21)$$

where q_1 is the position of the image due to surface 1. If the image due to surface 1 is virtual (Fig. 24a), q_1 is negative, and it is positive if the image is real (Fig. 24b). Now we apply Equation 20 to surface 2, taking $n_1 = n$ and $n_2 = 1$. (We make this switch in index because the light rays approaching surface 2 are in the material of the lens, and this material has index n .) Taking p_2 as the object distance for surface 2 and q_2 as the image distance give

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2} \quad (22)$$

We now introduce mathematically the fact that the image formed by the first surface acts as the

object for the second surface. We do this by noting from Figure 24 that p_2 , measured from surface 2, is related to q_1 as follows:

Virtual image from surface 1 (Fig. 24a): $p_2 = -q_1 + t$
(q_1 is negative)

Real image from surface 1 (Fig. 24b): $p_2 = -q_1 + t$
(q_1 is positive)

where t is the thickness of the lens. For a thin lens (one whose thickness is small compared to the radii of curvature), we can neglect t . In this approximation, we see that $p_2 = -q_1$ for either type of image from surface 1. (If the image from surface 1 is real, the image acts as a virtual object, so p_2 is negative.) Hence, Equation 22 becomes

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2} \quad (23)$$

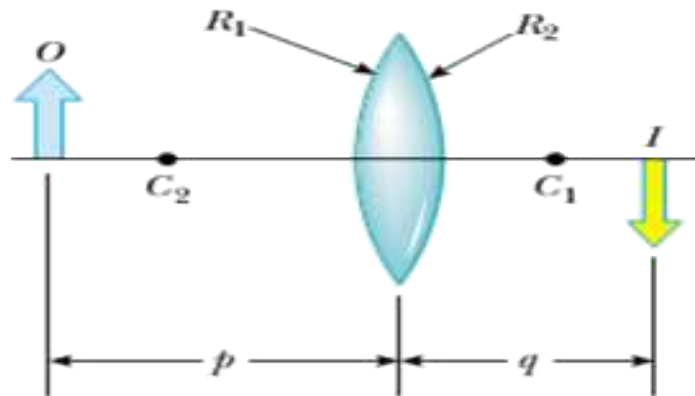


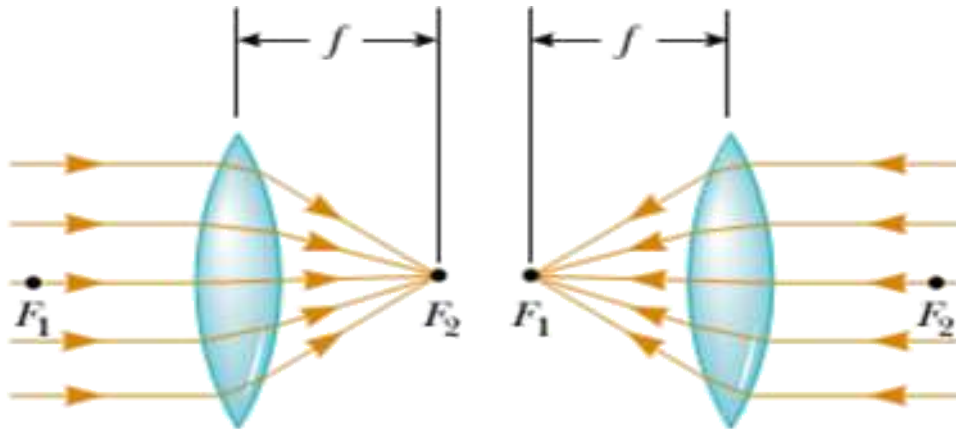
Fig.(25) Simplified geometry for a thin lens.

The inverse of *the focal length* f of a thin lens surrounded by air is given by the *lens makers' equation*:

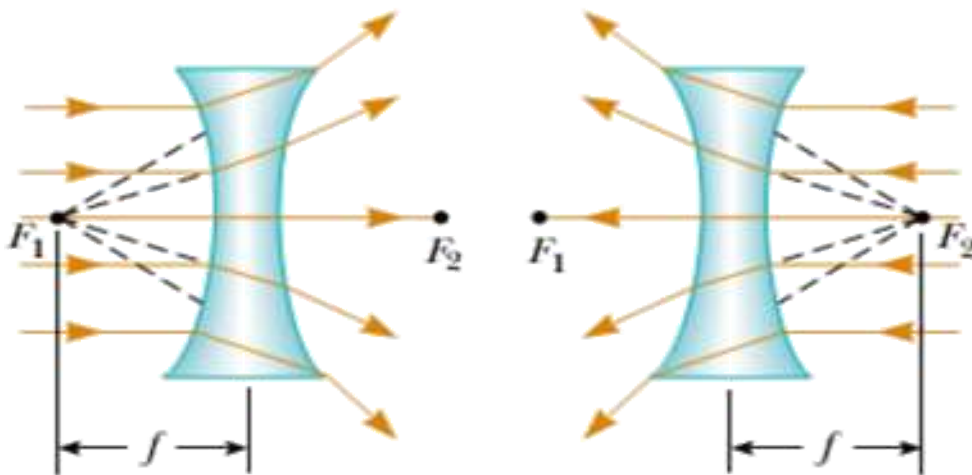
$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (26)$$

Converging lenses have positive focal lengths, and **diverging lenses** have negative focal lengths. For a thin lens, and in the paraxial ray approximation, the object and image distances are related by the *thin lens equation* :

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (27)$$



(a)



(b)

Fig.(24) (Left) Effects of a converging lens and a diverging lens on parallel rays. (Right) Parallel light rays pass through (a) a converging lens and (b) a diverging lens. The focal length is the same for light rays passing through a given lens in either direction. Both focal points F_1 and F_2 are the same distance from the lens.

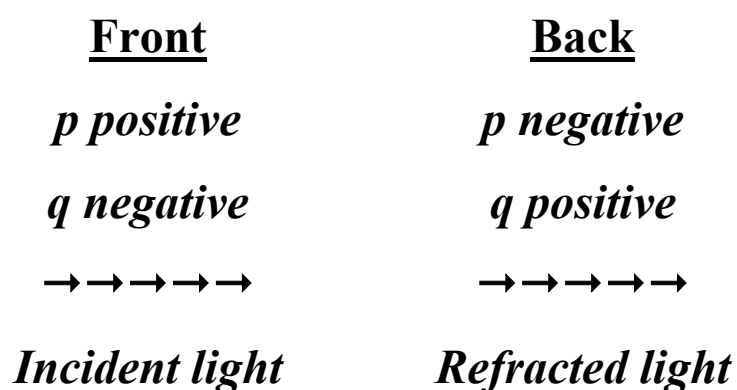


Fig.(25) A diagram for obtaining the signs of p and q for a thin lens. (This diagram also applies to a refracting surface.)

Figure25 is useful for obtaining the signs of p and q , and Table 2. gives the sign conventions for thin lenses. Note that these sign conventions are the same as those for refracting surfaces (see Table 1). Applying these rules to a biconvex lens, we see that when $p > f$,

Table 1.

Sign Conventions for Refracting Surfaces		
Quantity	Positive When	Negative When
Object location (p)	Object is in front of surface (real object)	Object is in back of surface (virtual object)
Image location (q)	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface

Table 2.

Sign Conventions for Thin Lenses		
Quantity	Positive When	Negative When
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

the quantities p , q , and R_1 are positive, and R_2 is negative. Therefore, p , q , and f are all positive when a converging lens forms a real image of an object. For a biconcave lens, p and R_2 are positive and q and R_1 are negative, with the result that f is negative .

Various lens shapes are shown in Figure 26a,b. Note that a converging lens is thicker at the center than at the edge, whereas a diverging lens is thinner at the center than at the edge .

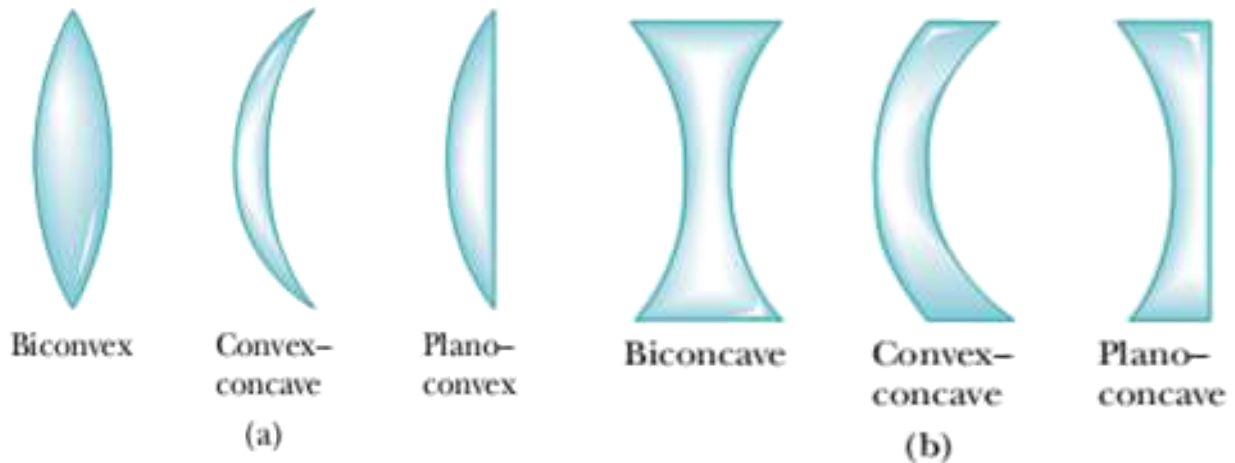


Fig.(26) Various lens shapes. (a) Converging lenses have a positive focal length and are thickest at the middle. (b) Diverging lenses have a negative focal length and are thickest at the edges.

14. Magnification of Images

Consider a thin lens through which light rays from an object pass. As with mirrors (Eq. 36.2) we could analyze a geometric construction to show that the lateral magnification of the image is

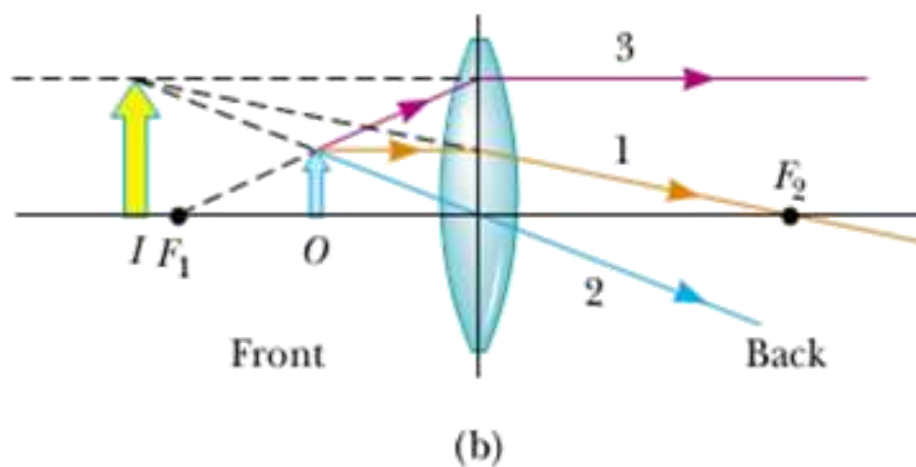
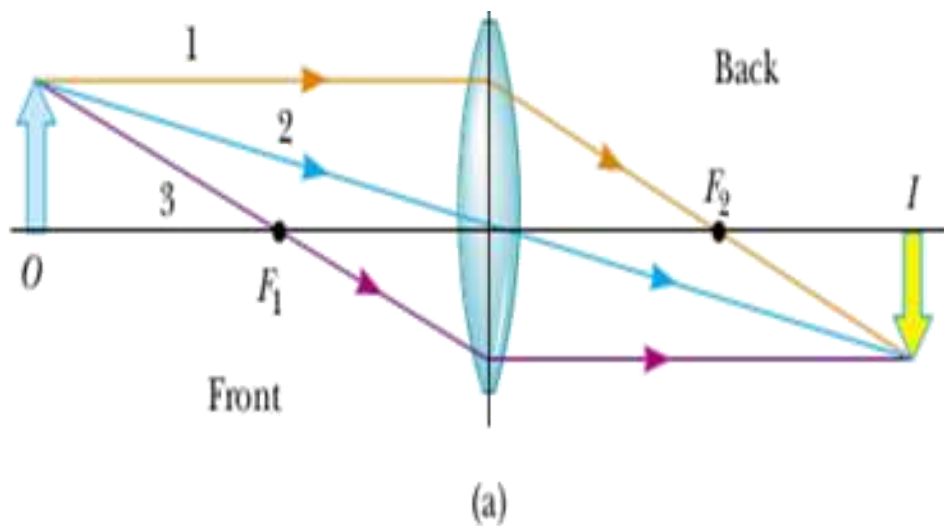
$$M = \frac{h'}{h} = -\frac{q}{p} \quad (28)$$

From this expression, it follows that when M is positive, the image is upright and on the same side of the lens as the object. When M is negative, the image is inverted and on the side of the lens opposite the object.

15. Ray Diagrams for Thin Lenses

To locate the image of a *converging lens* (Fig. 26 a and b), the following three rays are drawn from the top of the object:

- **Ray 1** is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- **Ray 2** is drawn through the center of the lens and continues in a straight line.
- **Ray 3** is drawn through the focal point on the front side of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.



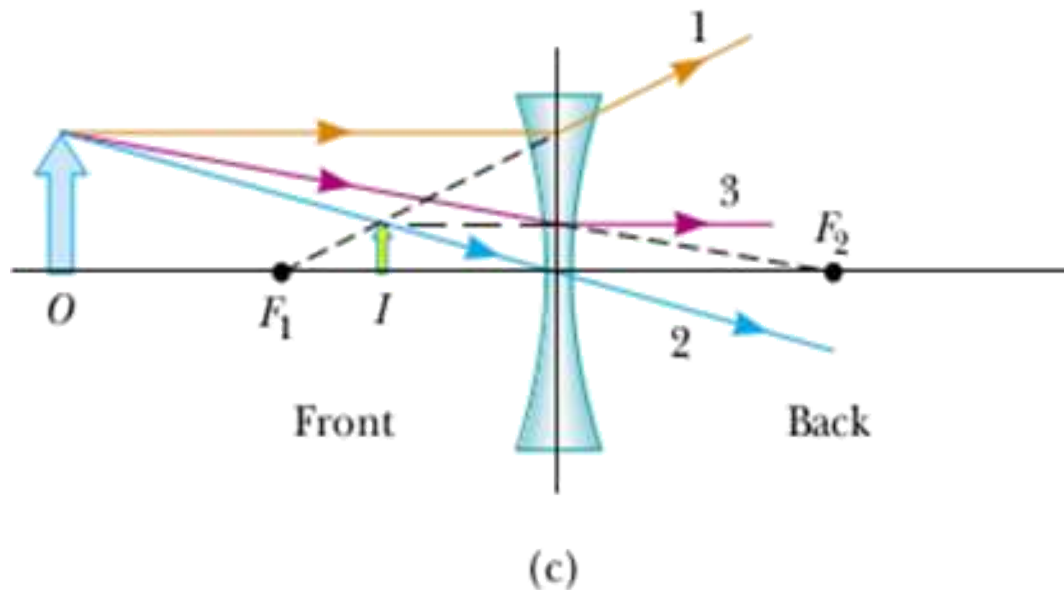


Fig.(26) Ray diagrams for locating the image formed by a thin lens. (a) When the object is in front of and outside the focal point of a converging lens, the image is real, inverted, and on the back side of the lens. (b) When the object is between the focal point and a converging lens, the image is virtual, upright, larger than the object, and on the front side of the lens. (c) When an object is anywhere in front of a diverging lens, the image is virtual, upright, smaller than the object, and on the front side of the lens.

To locate the image of a ***diverging lens*** (Fig. 26c), the following three rays are drawn from the top of the object:

- **Ray 1** is drawn parallel to the principal axis. After being refracted by the lens, this ray emerges directed away from the focal point on the front side of the lens.
- **Ray 2** is drawn through the center of the lens and continues in a straight line.
- **Ray 3** is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.

16. Combination of Thin Lenses

Let us consider the special case of a system of two lenses of focal lengths f_1 and f_2 in contact with each other. If $p_1 = p$ is the object distance for the combination, application of the thin lens equation (Eq. 36.16) to the first lens gives

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_1} \quad (29)$$

where q_1 is the image distance for the first lens. Treating this image as the object for the second lens, we see that the object distance for the second lens must be ($p_2 = -q_1$). The distances are the same because the lenses are in contact and assumed to be infinitesimally thin. The object distance is negative because the object is virtual.) Therefore, for the second lens,

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \quad (29')$$

$$- \frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_2} \quad (29'')$$

where $q = q_2$ is the final image distance from the second lens, which is the image distance for the combination. Adding the equations for the two lenses eliminates q_1 and gives

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2} \quad (29''')$$

If we consider replacing the combination with a single lens that will form an image at the same location, we see that its focal length is related to the individual focal lengths by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad (30)$$

Therefore, *two thin lenses in contact with each other are equivalent to a single thin lens having a focal length given by Equation 36.17*

17. Lens Aberrations

A precise analysis of image formation requires tracing each ray, using Snell's law at each refracting surface and the law of reflection at each reflecting surface. This procedure shows that the rays from a point object do not focus at a single point, with the result that the image is blurred. The departures of actual images from the ideal predicted by our simplified model are called ***aberrations***.

a. Spherical Aberrations

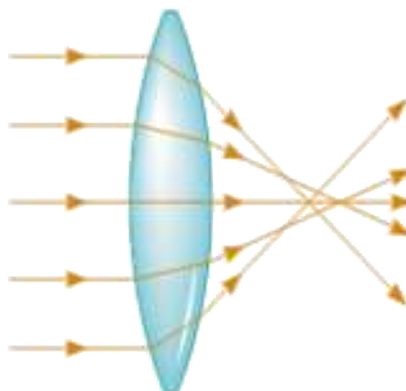


Fig.(27) Spherical aberration caused by a converging lens.
Does a diverging lens cause spherical aberration?

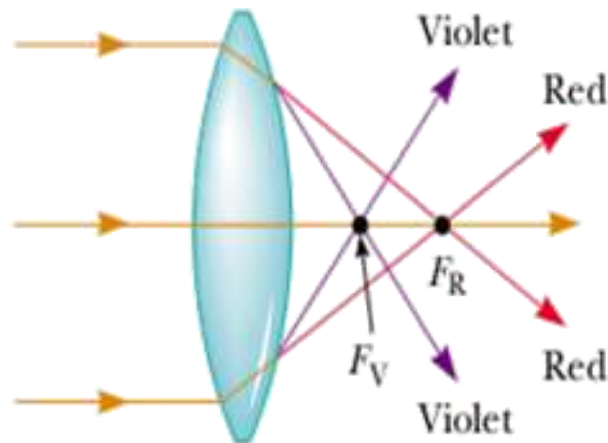


Fig.(28) Chromatic aberration caused by a converging lens.
Rays of different wavelengths focus at different points.

Spherical aberrations occur because the focal points of rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis. Figure 27 illustrates spherical aberration for parallel rays passing through a converging lens. Rays passing through points near the center of the lens are imaged farther from the lens than rays passing through points near the edges. Figure 27 showed a similar situation for a spherical mirror. In the case of mirrors, spherical aberration can be minimized through the use of a parabolic reflecting surface rather than a spherical surface.

b. Chromatic Aberrations

Different wavelengths of light refracted by a lens focus at different points gives rise to chromatic aberrations . we previously described how the index of refraction of a material varies with wavelength.

For instance, when white light passes through a lens, violet rays are refracted more than red rays (Fig. 28). From this we see that the focal length of a lens is greater for red light than for violet light. Other wavelengths (not shown in Fig. 28) have focal points intermediate between those of red and violet.

Chromatic aberration for a diverging lens also results in a shorter focal length for violet light than for red light, but on the front side of the lens. Chromatic aberration can be greatly reduced by combining a converging lens made of one type of glass and a diverging lens made of another type of glass.

18. The Camera

The photographic camera is a simple optical instrument , is shown in Figure 29 . a converging lens plays important rule inside the camera where the real images are constructed, and a film behind the lens to receive the image.

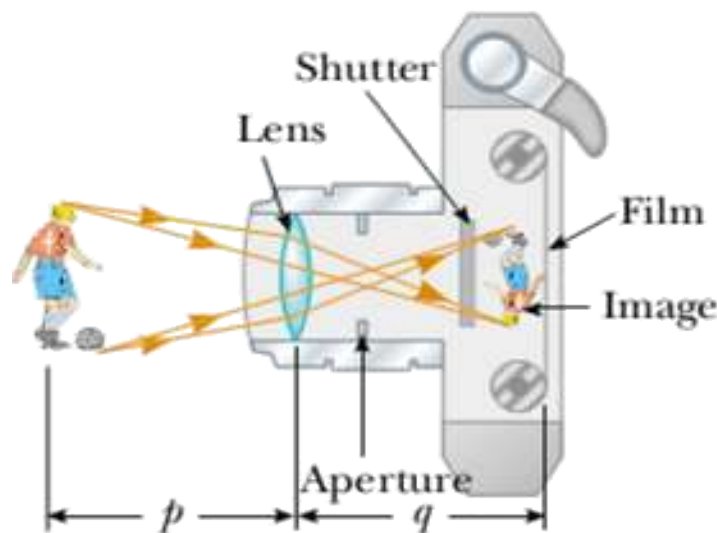


Fig.(29) Cross-sectional view of a simple camera. Note that in reality, $p \gg q$.

For proper focusing, which is necessary for the formation of sharp images, the lens-to-film distance depends on the object distance as well as on the focal length of the lens. The shutter, positioned behind the lens, is a mechanical device that is opened for selected time intervals, called *exposure times*. Typical shutter speeds (that is, exposure times) are (1/30)s, (1/60)s, (1/125)s, and (1/250)s that are to prevent the movement of the camera while the shutter is open, either short exposure times or a tripod should be used, even for stationary objects. The intensity I of the light reaching the film is proportional to the area of the lens, D^2 , and also is also proportional to $1/f^2$, and thus $I \propto D^2 / f^2$. The brightness of the image formed on the film depends on the light intensity, so we see that the image brightness depends on both the focal length and the diameter of the lens. The ratio f/D is called the ***f-number*** of a lens:

$$f\text{-number} \equiv \frac{f}{D} \quad (31)$$

Hence, the intensity of light incident on the film varies according to the following proportionality:

$$I \propto \frac{1}{\left(f/D\right)^2} \propto \frac{1}{(f\text{-number})^2} \quad (32)$$

since the *f-number* is often given as a description of the *lens speed*. Camera lens systems (that is, combinations of lenses with

adjustable apertures) are often marked with multiple *f-numbers*, usually *f/2.8*, *f/4*, *f/5.6*, *f/8*, *f/11*, and *f/16*. Any one of these settings can be selected by adjusting the aperture, which changes the value of *D*.

Example : The lens of a certain 35-mm camera (where 35mm is the width of the film strip) has a focal length of 55mm and a speed (an f-number) of *f/1.8*. The correct exposure time for this speed under certain conditions is known to be (1/500) s. a) Determine the diameter of the lens. b) Calculate the correct exposure time if the f-number is changed to *f/4* under the same lighting conditions.

Solution : a) From Equation 31 , we find that

$$D = \frac{f}{f\text{-number}} = \frac{55 \text{ mm}}{1.8} = 31 \text{ mm}$$

b) The total light energy hitting the film is proportional to the product of the intensity and the exposure time. If *I* is the light intensity reaching the film, then in a time interval Δt the energy per unit area received by the film is proportional to *I* Δt . Comparing the two situations, we require that $I_1 \Delta t_1 = I_2 \Delta t_2$, where Δt_1 is the correct exposure time for *f/1.8* and Δt_2 is the correct exposure time for *f/4*. Using this result together with Equation 36.19, we find that

$$\frac{\Delta t_1}{(f_1 - \text{number})^2} = \frac{\Delta t_2}{(f_2 - \text{number})^2}$$

$$\therefore \Delta t_2 = \left(\frac{f_2 - \text{number}}{f_1 - \text{number}} \right)^2 \Delta t_1 = \left(\frac{4}{1.8} \right)^2 \left(\frac{1}{500} \text{ s} \right) \approx \frac{1}{100} \text{ s}$$

As the aperture size is reduced, exposure time must increase.

19. The Eye

The mechanisms of eye view is like a camera where a normal eye focuses light and produces a sharp image. The eye controls the amount of light admitted and adjusts to produce correctly focused images which are far more complex, intricate, and effective than those in even the most sophisticated camera.

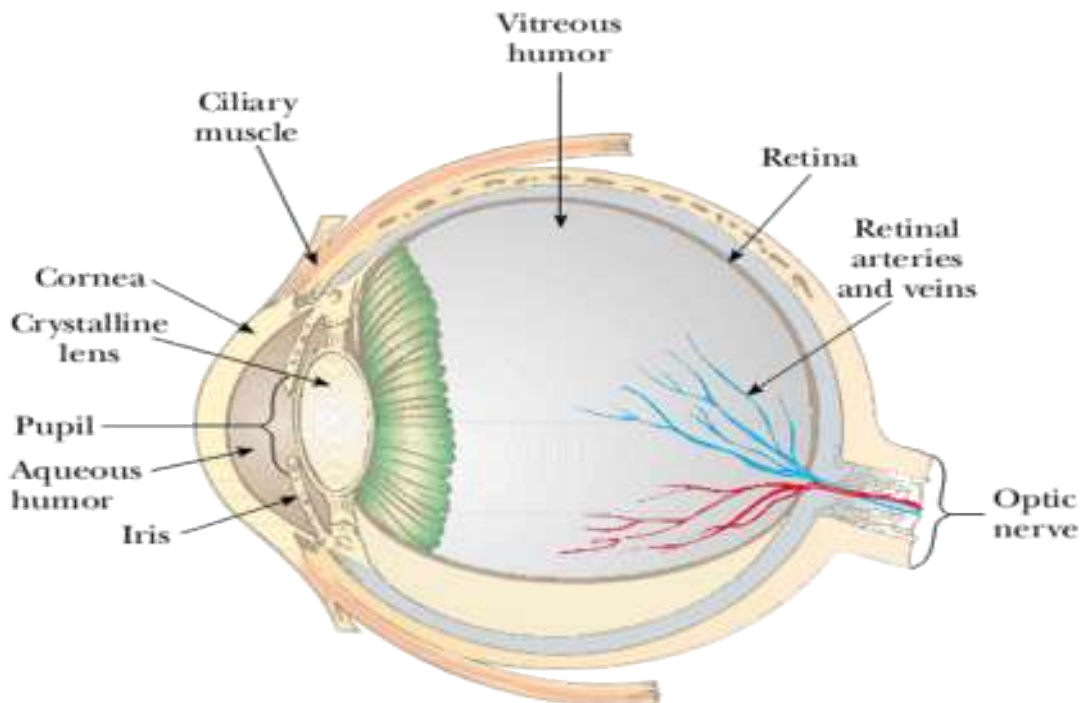


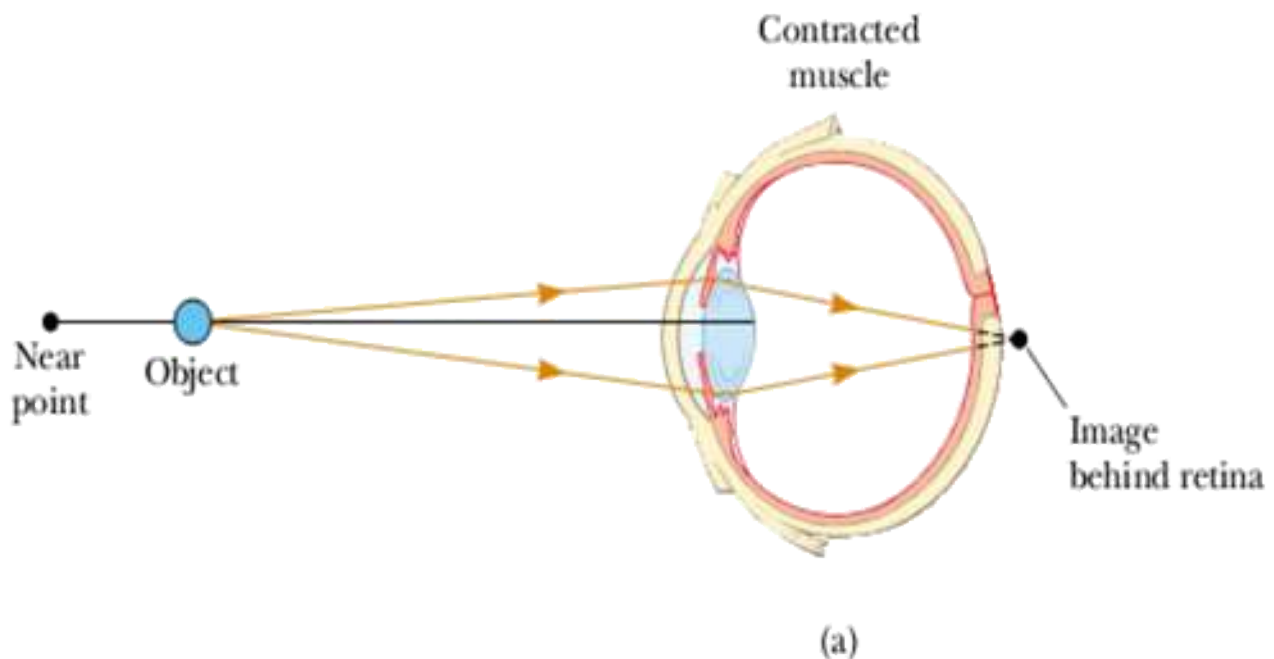
Fig.(30) Important parts of the eye.

The basic parts of the *human eye* are shown in Figure(30) . Light entering the eye passes through a transparent structure called *the cornea*, behind which are a clear liquid (*the aqueous humor*), a variable aperture (*the pupil*, which is an opening in the *iris*), and the *crystalline lens*. Most of the refraction occurs at the outer surface of the eye, where the *cornea* is covered with a film of tears. Relatively little refraction occurs in the crystalline lens because the *aqueous humor* in contact with the lens has an average *index of refraction* close to that of the lens. The *f-number range* of the eye is from about *f/2.8* to *f/16*. *The cornea*–lens system focuses light onto the back surface of the eye, the *retina*, which consists of millions of sensitive receptors called *rods* and *cones*. When stimulated by light, these receptors send impulses via the *optic nerve* to *the brain*, where an image is perceived. By this process, a distinct image of an object is observed when the image falls on the retina.

a. Accommodation : The eye focuses on an object by varying the shape of the pliable crystalline lens through an amazing process. An important component of accommodation is the ciliary muscle, which is situated in a circle around the rim of the lens. Thin filaments, called *zonulas*, run from this muscle to the edge of the lens. When the eye is focused on a distant object, the ciliary muscle

is relaxed, tightening the **zonulas** that attach the muscle to the edge of the lens. Accommodation is limited in that objects very close to the eye produce blurred images. The near point is the closest distance for which the lens can accommodate to focus light on the retina. This distance usually increases with age and has an average value of 25cm. Typically, at age 10 the near point of the eye is about 18cm. It increases to about 25cm at age 20, to 50cm at age 40, and to 500cm or greater at age 60. The far point of the eye represents the greatest distance for which the lens of the relaxed eye can focus light on the retina. A person with normal vision can see very distant objects and thus has a far point that can be approximated as infinity.

b. Conditions of the Eye :



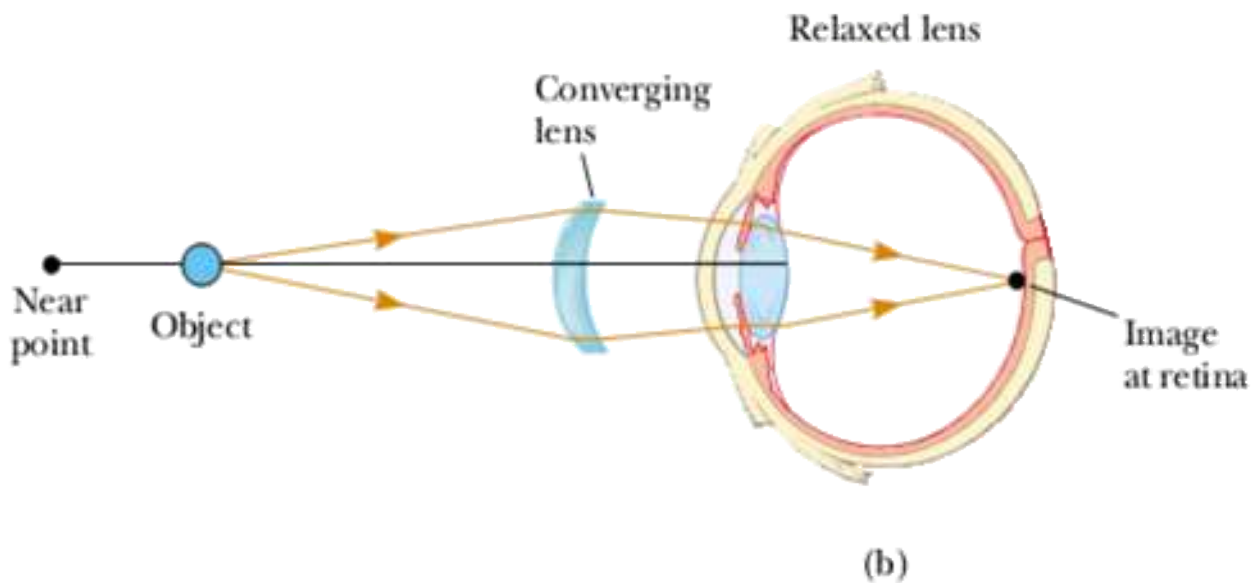


Fig.(31) (a) When a farsighted eye looks at an object located between the near point and the eye, the image point is behind the retina, resulting in blurred vision. The eye muscle contracts to try to bring the object into focus. (b) Farsightedness is corrected with a converging lens.

When the eye suffers a mismatch between the focusing range of the *lens–cornea* system and the length of the eye, with the result that light rays from a near object reach the retina before they converge to form an image, as shown in Figure 31a, the condition is known as *farsightedness*(or *hyperopia*). The condition can be corrected by placing a converging lens in front of the eye, as shown in Figure 31b. The lens refracts the incoming rays more toward the principal axis before entering the eye, allowing them to converge and focus on the retina.

A person with nearsightedness(or myopia), another mismatch condition, can focus on nearby objects but not on faraway objects.

The far point of the nearsighted eye is not infinity and may be less than 1m. The **maximum focal length** of the near-sighted eye is insufficient to produce a sharp image on the retina, and rays from a distant object converge to a focus in front of the retina. They then continue past that point, diverging before they finally reach the retina and causing blurred vision

Nearsightedness can be corrected with a diverging lens, as shown in Figure 32a,b. The lens refracts the rays away from the principal axis before they enter the eye, allowing them to focus on the retina.

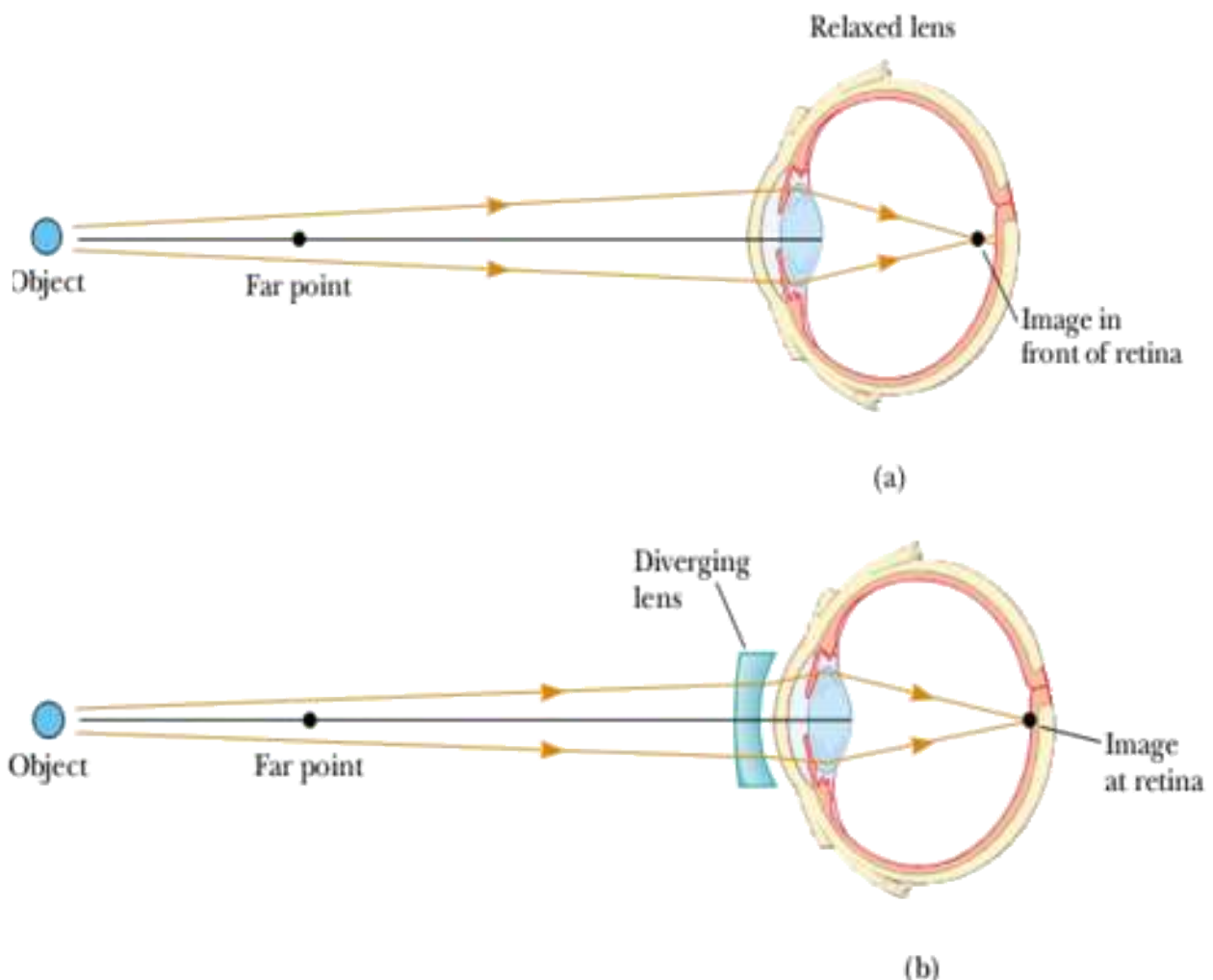


Fig.(32) (a) When a nearsighted eye looks at an object that lies beyond the eye's far point, the image is formed in front of the retina, resulting in blurred vision. (b) Nearsightedness is corrected with a diverging lens

c. The eye defect "astigmatism":

Light from a point source produces a line image on the retina. This condition arises when either the cornea or the lens or both are not perfectly symmetric. Astigmatism can be corrected with lenses that have different curvatures in two mutually perpendicular directions.

Optometrists and ophthalmologists usually prescribe lenses measured in ***diopters***: the ***power P*** of a lens in diopters equals the inverse of the ***focal length*** in meters: **$P = 1/f$** . For example, a converging lens of focal length +20cm has a power of +5.0 diopters, and a diverging lens of focal length -40cm has a power of -2.5 diopters.

d. The Simple Magnifier :

The simple magnifier consists of a single converging lens. As the name implies, this device increases the apparent size of an object. The size of the image formed at the retina depends on the angle θ sub-tended by the object at the eye. As the object moves closer to the eye, θ increases and a larger image is observed. However, an average normal eye cannot focus on an object closer than about 25cm, the near point (Fig. 33a). Therefore, θ is maximum at the near point.

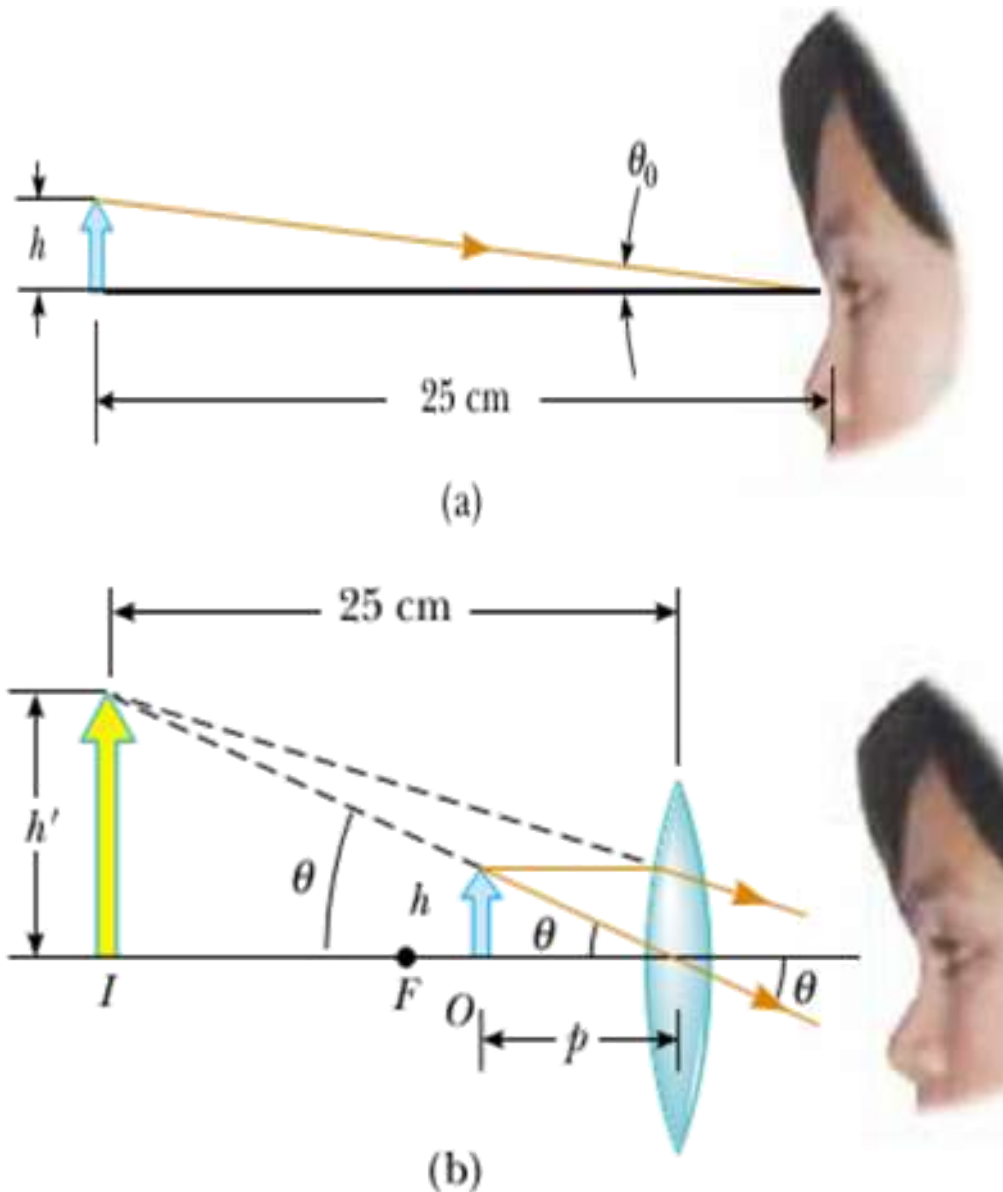


Fig.(33) (a) An object placed at the near point of the eye ($p = 25\text{cm}$) subtends an angle $\theta_0 \approx h/25$ at the eye. (b) An object placed near the focal point of a converging lens produces a magnified image that subtends an angle $\theta \approx h'/25$ at the eye.

To increase the apparent angular size of an object, a converging lens can be placed in front of the eye as in Figure 33b, with the object located at point O , just inside the focal point of the lens. At

this location, the lens forms a virtual, upright, enlarged image. We define **angular magnification m** as the ratio of the angle subtended by an object with a lens in use (angle θ in Fig. 33b) to the angle subtended by the object placed at the near point with no lens in use (angle θ_o in Fig.33a):

$$m \equiv \frac{\theta}{\theta_o} \quad (33)$$

The angular magnification is a maximum when the image is at the near point of the eye—that is, when $q = -25\text{cm}$. The object distance corresponding to this image distance can be calculated from the thin lens equation:

$$\frac{1}{p} + \frac{1}{-25\text{cm}} = \frac{1}{f}$$

$$p = \frac{25f}{25 + f}$$

where f is the focal length of the magnifier in centimeters. If we make the small-angle approximations

$$\tan \theta_o \approx \theta_o \approx \frac{h}{25} \quad \text{and} \quad \tan \theta \approx \theta \approx \frac{h}{p} \quad (34)$$

Equation 33 becomes

$$m_{max} = \frac{\theta}{\theta_o} = \frac{h/p}{h/25} = \frac{25}{p} = \frac{25}{25f/(25+f)}$$

$$m_{max} = 1 + \frac{25cm}{f}$$

For the image formed by the magnifying lens to appear at infinity, the object has to be at the focal point of the lens. In this case, Equations 34 become

$$\theta_o \approx \frac{h}{25} \quad \text{and} \quad \theta \approx \frac{h}{f}$$

and the magnification is

$$m_{min} = \frac{\theta}{\theta_o} = \frac{25cm}{f}$$

With a single lens, it is possible to obtain angular magnifications up to about 4 without serious aberrations. Magnifications up to about 20 can be achieved by using one or two additional lenses to correct for aberrations .

Chapter II

ELECTRICITY
&
MAGNETISM

Chapter II (a) "Electricity"

Electricity is the set of physical phenomena associated with the presence and flow of electric charge. Electricity gives a wide variety of well-known effects, such as lightning, static electricity, electromagnetic induction and electric current. In addition, electricity permits the creation and reception of electromagnetic radiation such as radio waves.

In electricity, charges produce electromagnetic fields which act on other charges. Electricity occurs due to several types of physics:

1. Electric charge:

A property of some subatomic particles(subatomic particles are particles much smaller than atoms. There are two types of subatomic particles: elementary particles, which according to current theories are not made of other particles; and composite particles.), which determines their electromagnetic interactions. Electrically charged matter is influenced by, and produces, electromagnetic fields. Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. There are two types of electric charges: positive and negative. Positively charged substances are repelled from other positively charged substances, but attracted to

negatively charged substances; negatively charged substances are repelled from negative and attracted to positive. An object is negatively charged if it has an excess of electrons, and is otherwise positively charged or uncharged.

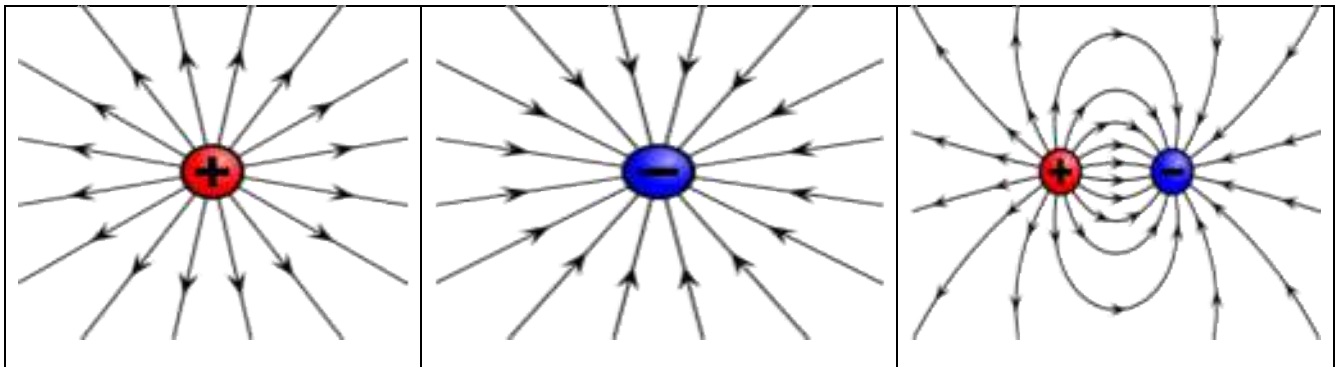


Fig.(1)

The SI derived unit of electric charge is the coulomb (C), although in electrical engineering it is also common to use the ampere-hour (Ah), and in chemistry it is common to use the elementary charge (e) as a unit. The symbol Q is often used to denote charge. The early knowledge of how charged substances interact is now called classical electrodynamics, and is still very accurate if quantum effects do not need to be considered.

The electric charge is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction. Electrically charged matter is influenced by, and produces, electromagnetic fields. The interaction between a

moving charge and an electromagnetic field is the source of the electromagnetic force, which is one of the four fundamental forces.

Electric charges have the following important properties:

- Charges of opposite sign attract one another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

It is known that the **Conductors** are materials in which electrons move freely while the **Insulators** are materials in which electrons do not move freely. By applying **Coulomb's law** states that *the electric force exerted by a charge q_1 on a second charge q_2 is*

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} \quad (1)$$

where r is the distance between the two charges and $\hat{\mathbf{r}}$ is a unit vector directed from $\mathbf{q}_1$ toward $\mathbf{q}_2$. The constant k_e which is called the **Coulomb constant**, has the value $k_e = 8.99 \times 10^9 \text{ N.m}^2 / \text{C}^2$. The smallest unit of free charge e known to exist in nature is the charge on an electron ($-e$) or proton ($+e$), where $e = 1.60219 \times 10^{-19} \text{ C}$.

2. The electric field E

An especially simple type of electromagnetic field produced by an electric charge even when it is not moving (i.e., there is no electric current). The electric field produces a force on other charges in its vicinity. Electric field lines are useful for visualizing the electric field. Field lines begin on positive charge and terminate on negative charge. Electric field lines are parallel to the direction of the electric field, and the density of these field lines is a measure of the magnitude of the electric field at any given point.

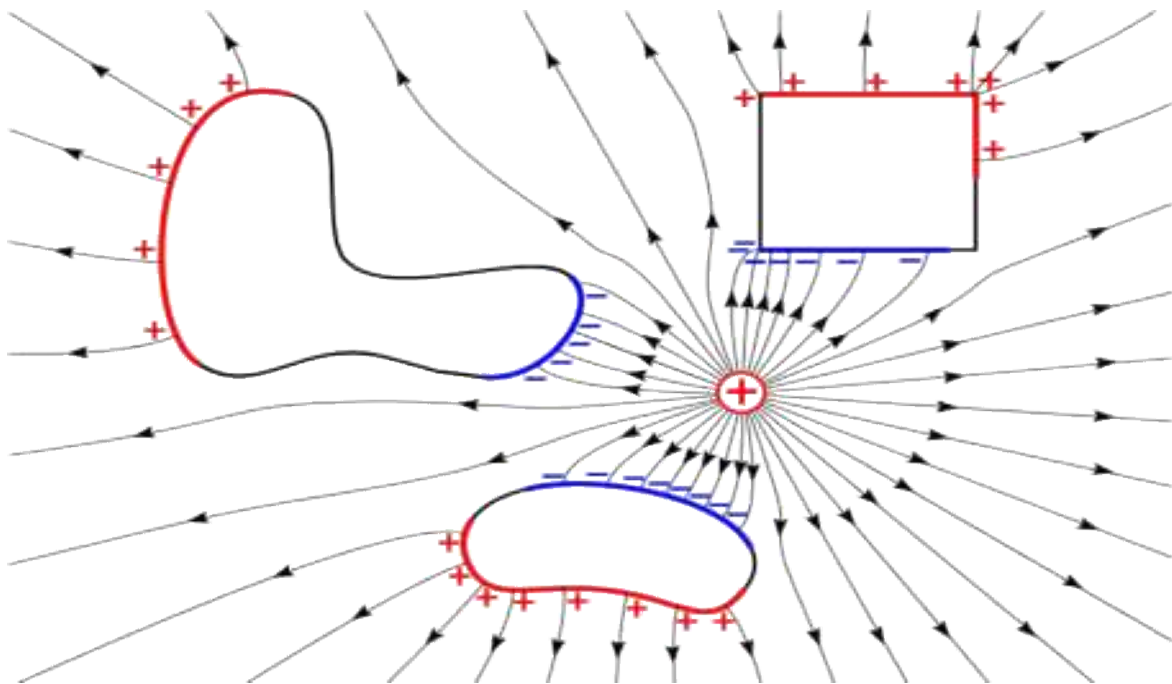


Fig.(2) The electrostatic field (*lines with arrows*) of a nearby positive charge (+) causes the mobile charges in conductive objects to separate due to electrostatic induction. Negative charges (*blue*) are attracted and move to the surface of the object facing the external charge. Positive charges (*red*) are repelled and move to the surface facing away. These induced surface charges are exactly the right size and shape so their opposing electric field cancels the electric field of the external charge

throughout the interior of the metal. Therefore the electrostatic field everywhere inside a conductive object is zero, and the electrostatic potential is constant.

Definition " *The electric field E at a given point is defined as the (vectorial) force F that would be exerted on a stationary test particle of unit charge by electromagnetic forces (i.e. the Lorentz force). A particle of charge q would be subject to a force $F = q \cdot E$.* "

At a distance r from a point charge q , the electric field due to the charge is given by

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \quad (2)$$

where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the point in question. The electric field is directed radially outward from a positive charge and radially inward a negative charge. Its SI units are newtons per coulomb ($\text{N} \cdot \text{C}^{-1}$) or, equivalently, volts per metre ($\text{V} \cdot \text{m}^{-1}$), which in terms of SI base units are $\text{kg} \cdot \text{m} \cdot \text{s}^{-3} \cdot \text{A}^{-1}$.

The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \quad (3)$$

The electric field at some point due to a continuous charge distribution is

$$\mathbf{E} = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \quad (4)$$

where dq is the charge on one element of the charge distribution and r is the distance from the element to the point in question. Electric field lines describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of $\mathbf{E}$ in that region. A charged particle of mass m and charge q moving in an electric field $\mathbf{E}$ has an acceleration

$$\mathbf{a} = \frac{q \mathbf{E}}{m} \quad (5)$$

a. Charge density $\rho(r)$:

In electromagnetism, **charge density** is a measure of electric charge per unit volume of space, in one, two or three dimensions. More specifically: the **linear** (λ_q), **surface** (σ_q), or **volume** (ρ_q) **charge density** is the amount of electric charge per unit length, surface area, or volume, respectively. The respective SI

units are $\text{C}\cdot\text{m}^{-1}$, $\text{C}\cdot\text{m}^{-2}$ or $\text{C}\cdot\text{m}^{-3}$. **Charge density** can depend on position, and charge and thus charge density can be negative. It should not be confused with the charge carrier density, the number of charge carriers (e.g. electrons, ions) in a material per unit volume, not including the actual charge on the carriers .

b. Continuous charges :

The linear charge density is the ratio of an infinitesimal electric charge dQ where Q is the total charge of region (SI unit: C) to an infinitesimal line element $d\ell$,

$$\lambda_q = \frac{dQ}{dl} , \quad \Rightarrow \quad Q = \int \lambda_q(r) dl \quad (6)$$

similarly the surface charge density uses a surface area element dS

$$\sigma_q = \frac{dQ}{dS} , \quad \Rightarrow \quad Q = \int_S \sigma_q(r) dS \quad (7)$$

and the volume charge density uses a volume element dV

$$\rho_q = \frac{dQ}{dV} , \quad \Rightarrow \quad Q = \int_V \rho_q(r) dV \quad (8)$$

where the subscript q is to clarify that the density is for electric charge, not other densities like mass density, number density, probability density, and prevent conflict with the many other uses of λ , σ , ρ in electromagnetism for wavelength, electrical resistivity and conductivity.

3. Electric flux

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle with the normal to a surface of area A , the electric flux through the surface is

$$\Phi_E = EA \cos \theta \quad (9)$$

In general, the electric flux through a surface is

$$\Phi_E = \int_{\text{surface}} E \cdot dA \quad (10)$$

Table(1)

Typical Electric Field Calculations Using Gauss's Law		
Charge Distribution	Electric Field	Location
Insulating sphere of radius R , uniform charge density, and total charge Q	$\begin{cases} k_e \frac{Q}{r^2} \\ k_e \frac{Q}{R^2} r \end{cases}$	$\begin{cases} r > R \\ r < R \end{cases}$
Thin spherical shell of radius R and total charge Q	$\begin{cases} k_e \frac{Q}{r^2} \\ 0 \end{cases}$	$\begin{cases} r > R \\ r < R \end{cases}$
Line charge of infinite length and charge per unit length λ	$2k_e \frac{\lambda}{r}$	Outside the line
Infinite charged plane having surface charge density σ	$\frac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\begin{cases} \frac{\sigma}{\epsilon_0} \\ 0 \end{cases}$	$\begin{cases} \text{Just outside the conductor} \\ \text{Inside the conductor} \end{cases}$

You should be able to apply Equations 9 and 10 in a variety of situations, particularly those in which symmetry simplifies the calculation. **Gauss's law** says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge q inside the surface divided by ϵ_0 :

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0} \quad (11)$$

Gauss's law

Gauss's law states that " the total electric flux through any closed surface in free space of any shape drawn in an electric field is proportional to the total electric charge enclosed by the surface."

Mathematically, Gauss's law takes the form of an integral equation

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{enclosed} = \int_V \frac{\rho}{\epsilon_0} \cdot d^3 r,$$

where $d^3 r = dx \, dy \, dz$ is a volume element. If the charge is distributed over a surface or along a line, replace by σdA or λdl .

The Divergence Theorem allows Gauss's Law to be written in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

where $\vec{\nabla}$ is the divergence operator .

Using Gauss's law, you can calculate the electric field due to various symmetric charge distributions. Table(1) lists some typical results. A conductor in electrostatic equilibrium has the following properties

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on the conductor resides entirely on its surface.
3. The electric field just outside the conductor is perpendicular to its surface and has a
4. magnitude σ/ϵ_0 , where σ is the surface charge density at that point.
5. On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest .

4. Electric Potential

When a positive test charge q_0 is moved between points A and B in an electric field $\mathbf{E}$, the change in the potential energy of the charge–field system is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \quad (12)$$

The electric potential $V = U/q_0$ is a scalar quantity and has the units of J/C, where $1 \text{ J/C} \equiv 1 \text{ V}$.

The potential difference ΔV between points A and B in an electric field $\mathbf{E}$ is defined as

$$\Delta V \equiv \frac{\Delta U}{q_0} = - \int_A^B \mathbf{E} \cdot d\mathbf{s} \quad (13)$$

The potential difference between two points A and B in a uniform electric field $\mathbf{E}$, where $\mathbf{s}$ is a vector that points from A to B and is parallel to $\mathbf{E}$ is

$$\Delta V = -Ed \quad (14)$$

where $d = |\mathbf{s}|$. An **equipotential surface** is one on which all points are at the same electric potential. **Equipotential surfaces** are perpendicular to electric field lines. If we define $V = 0$ at $\mathbf{r}_A = \infty$, the **electric potential** due to a point charge at any distance $\mathbf{r}$ from the charge is

$$V = k_e \frac{q}{r} \quad (15)$$

We can obtain the electric potential associated with a group of point charges by summing the

potentials due to the individual charges. The potential energy associated with a pair of point charges

separated by a distance $\mathbf{r}_{12}$ is

$$E_x = -\frac{dV}{dx} \quad (16)$$

The electric potential due to a continuous charge distribution is

$$V = k_e \int \frac{dq}{r} \quad (17)$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric

potential. The potential is constant everywhere inside the conductor and equal to its value at the surface. Table 2. lists electric potentials due to several charge distributions.

Electric Potential Due to Various Charge Distributions		
Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma [(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, <i>insulating</i> solid sphere of radius R and total charge Q	$\begin{cases} V = k_e \frac{Q}{r} \\ V = \frac{k_e Q}{2R} \left(3 - \frac{r^2}{R^2} \right) \end{cases}$	$r \geq R$ $r < R$
Isolated <i>conducting</i> sphere of radius R and total charge Q	$\begin{cases} V = k_e \frac{Q}{r} \\ V = k_e \frac{Q}{R} \end{cases}$	$r > R$ $r \leq R$

5. Capacitance and Dielectrics

A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance of any capacitor is the ratio of the charge **Q** on either conductor to the potential difference ΔV between them:

$$C = \frac{Q}{\Delta V} \quad (18)$$

The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference. The SI unit of capacitance is coulombs per volt, or the *farad (F)*, and $1F = 1C/V$. Capacitance expressions for various geometries are summarized in Table 3. . If two or more capacitors are connected in parallel, then the potential difference is the same across all of them. The equivalent capacitance of a parallel combination of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \dots \quad (19)$$

If two or more capacitors are connected in series, the charge is the same on all of them, and the equivalent capacitance of the series combination is given by

Table 3.

Capacitance and Geometry		
Geometry	Capacitance	Equation
Isolated sphere of radius R (second spherical conductor assumed to have infinite radius)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \frac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b , respectively	$C = \frac{\ell}{2k_e \ln(b/a)}$	26.4
Spherical capacitor with inner and outer radii a and b , respectively	$C = \frac{ab}{k_e (b - a)}$	26.6

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \quad (20)$$

These two equations enable you to simplify many electric circuits by replacing multiple capacitors with a single equivalent capacitance.

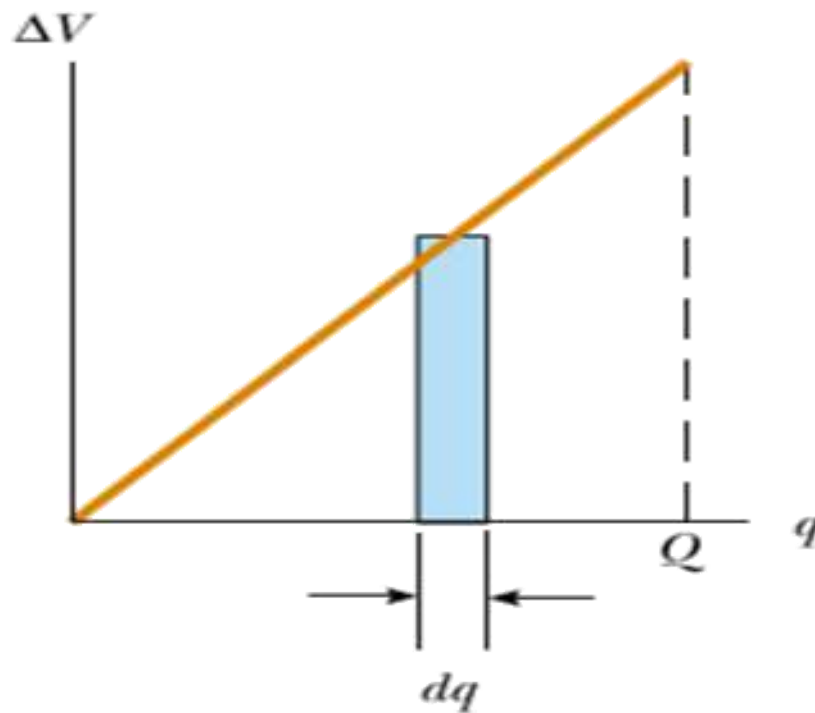


Fig.(6) A plot of potential difference versus charge for a capacitor is a straight line having a slope $1/C$. The work required to move charge dq through the potential difference ΔV existing at the time across the capacitor plates is given approximately by the area of the shaded rectangle. The total work required to charge the capacitor to a final charge Q is the triangular area under the straight line, $W = 1/2 Q\Delta V$. Don't forget that $1V=J/C$; hence, the unit for the triangular area is the joule.

To calculate the **stored energy** inside any capacitor , we suppose that q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. It is known that the work necessary to transfer an increment of charge dq from the plate carrying charge $-q$ to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V dq = \frac{q}{C} dq \quad (21)$$

This is illustrated in Figure 6 . The total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C} \quad (22)$$

The work done in charging the capacitor appears as electric potential energy U stored in the capacitor. Using Equation 22 , we can express the potential energy stored in a charged capacitor with charge Q in the following forms:

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 \quad (23)$$

The *stored energy* inside any capacitor can be defined as "*Energy is stored in a capacitor because the charging process is equivalent to the transfer of charges from one conductor at a lower electric potential to another conductor at a higher potential.*" When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a dimensionless factor **1**, called the dielectric constant:

$$C = k C_o \quad (24)$$

where C_0 is the capacitance in the absence of the dielectric. The increase in capacitance is due to a decrease in the magnitude of the electric field in the presence of the dielectric. The decrease in the magnitude of E arises from an internal electric field produced by aligned dipoles in the dielectric.

The **electric dipole moment** p of an electric dipole has a magnitude

$$P \equiv 2 a q \quad (23)$$

The direction of the electric dipole moment vector is from the negative charge toward the positive charge. The torque acting on an electric dipole in a uniform electric field E is

$$\tau = P \times E \quad (24)$$

The potential energy of the system of an electric dipole in a uniform external electric field E is

$$U = -P.E \quad (25)$$

6. Current and Resistance

We study the flow of electric charges through a piece of material. The amount of flow depends on the material through which the charges are passing and the potential difference across the material.

Whenever there is a net flow of charge through some region, an electric current is said to exist.

To define current more precisely, suppose that charges are moving perpendicular to a surface of area A , as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.)

. The current is the rate at which charge flows through this surface. If ΔQ is the amount of charge

that passes through this area in a time interval Δt , the **average current** I_{av} is equal to the charge that

passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t} \quad (26)$$

If the rate at which charge flows varies in time, then the current varies in time; we define *the electric current I in a conductor which has related the instantaneous current I as the differential limit of average current "* as:

$$I \equiv \frac{dQ}{dt} \quad (27)$$

where dQ is the charge that passes through a cross section of the conductor in a time interval dt . The SI unit of current is the **ampere(A)**, where $1 \text{ A} = 1\text{C/s}$. The average current in a conductor

is related to the motion of the charge carriers through the relationship

$$I_{av} = nq v_d A \quad (28)$$

where n is the density of charge carriers, q is the charge on each carrier, v_d is the drift speed, and A is the cross-sectional area of the conductor. The magnitude of the current density J in a conductor is the current per unit area:

$$J = \frac{I}{A} = nq v_d \quad (29)$$

The current density in an ohmic conductor is proportional to the electric field according to the expression

$$\mathbf{J} = \sigma \mathbf{E} \quad (30)$$

The proportionality constant σ is called the conductivity of the material of which the conductor is made. The inverse of σ is known as resistivity ρ (that is, $\rho = 1/\sigma$). Equation 30 is known as Ohm's law, and a material is said to obey this law if the ratio of its current density $\mathbf{J}$ to its applied electric field $\mathbf{E}$ is a constant that is independent of the applied field.

The resistance R of a conductor is defined as

$$R \equiv \frac{\Delta V}{I} \quad (31)$$

where ΔV is the potential difference across it, and I is the current it carries. The SI unit of *resistance* is *volts* per *ampere*, which is defined to be 1 ohm (Ω) ; that is, $1\Omega = 1\text{V/A}$. If the resistance is independent of the applied potential difference, the conductor obeys Ohm's law. For a uniform block of material of cross sectional area and length ℓ , the resistance over the length ℓ , is

$$R = \rho \frac{\ell}{A} \quad (32)$$

where ρ is the resistivity of the material. In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a drift velocity v_d that is opposite the electric field and given by the expression

$$v_d = \frac{q E}{m_e} \tau \quad (33)$$

where τ is the average time interval between electron–atom collisions, m_e is the mass of the electron, and q is its charge. According to this model, the resistivity of the metal is

$$\rho = \frac{m_e}{n q^2 \tau} \quad (34)$$

where n is the number of free electrons per unit volume. The resistivity ρ of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad (35)$$

where α is the temperature coefficient of resistivity and ρ_0 is the resistivity at some reference temperature T_0 . If a potential difference ΔV is maintained across a circuit element, the power, or rate at which energy is supplied to the element, is

$$\mathcal{P} = I \Delta V \quad (36)$$

Because the potential difference across a resistor is given by $\Delta V = IR$, we can express the power delivered to a resistor in the form

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R} \quad (37)$$

The energy delivered to a resistor by electrical transmission appears in the form of internal energy in the resistor .

7. Direct Current Circuits

The emf of a battery is equal to the voltage across its terminals when the current is zero. That is, the emf is equivalent to the open-circuit voltage of the battery . The equivalent resistance of a set of resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \dots \quad (38)$$

The equivalent resistance of a set of resistors connected in parallel is found from the relationship .

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \quad (39)$$

If it is possible to combine resistors into series or parallel equivalents, the preceding two equations make it easy to determine how the resistors influence the rest of the circuit .

Circuits involving more than one loop are conveniently analyzed with the use of Kirchhoff's rules:

1. Junction rule. The sum of the currents entering any junction in an electric circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out} \quad (40)$$

2. Loop rule. The sum of the potential differences across all elements around any circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0 \quad (41)$$

The first rule is a statement of conservation of charge; the second is equivalent to a statement of conservation of energy.

When a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$. When a resistor is traversed in the direction opposite the current, $\Delta V = +IR$. When a source of emf is traversed in the direction of the emf (negative terminal to positive terminal), the potential difference is $+\mathcal{E}$. When a source of emf is traversed opposite the emf (positive to negative), the potential difference is $-\mathcal{E}$. The use of these rules together with Equations 40 and 41 allows you to analyze electric circuits. If a capacitor is charged with a battery through a resistor of resistance R , the charge on the capacitor and the current in the circuit vary in time according to the expression

$$q(t) = Q (1 - e^{-t/RC}) \quad (42)$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \quad (43)$$

where $Q = C\mathcal{E}$ is the maximum charge on the capacitor. The product RC is called the **time constant** τ of the circuit. If a charged capacitor is discharged through a resistor of resistance R , the charge and current decrease exponentially in time according to the expressions

$$q(t) = Q e^{-t/RC} \quad (44)$$

$$I(t) = -I_0 e^{-t/RC} \quad (45)$$

where Q is the initial charge on the capacitor and $I_0 = Q/RC$ is the initial current in the circuit .

Chapter II b "Magnetism"

8. Magnetic Fields

Historically, the symbol $\mathbf{B}$ has been used to represent a magnetic field, and this is the notation we use in this text. The direction of the magnetic field $\mathbf{B}$ at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with magnetic field lines.

We can define a magnetic field $\mathbf{B}$ at some point in space in terms of the magnetic force $\mathbf{F}_B$ that the field exerts on a charged particle moving with a velocity $\mathbf{v}$, which we call the test object. For the time being, let us assume that no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

- ☒ The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.

- ☒ The magnitude and direction of F_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field $\mathbf{B}$.
- ☒ When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- ☒ When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\mathbf{v}$ and $\mathbf{B}$; that is, $\mathbf{F}_B$ is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$ (Fig.1a).
- ☒ The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig.1b)
- ☒ The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin \theta$, where θ is the angle the particle's velocity vector makes with the direction of $\mathbf{B}$.

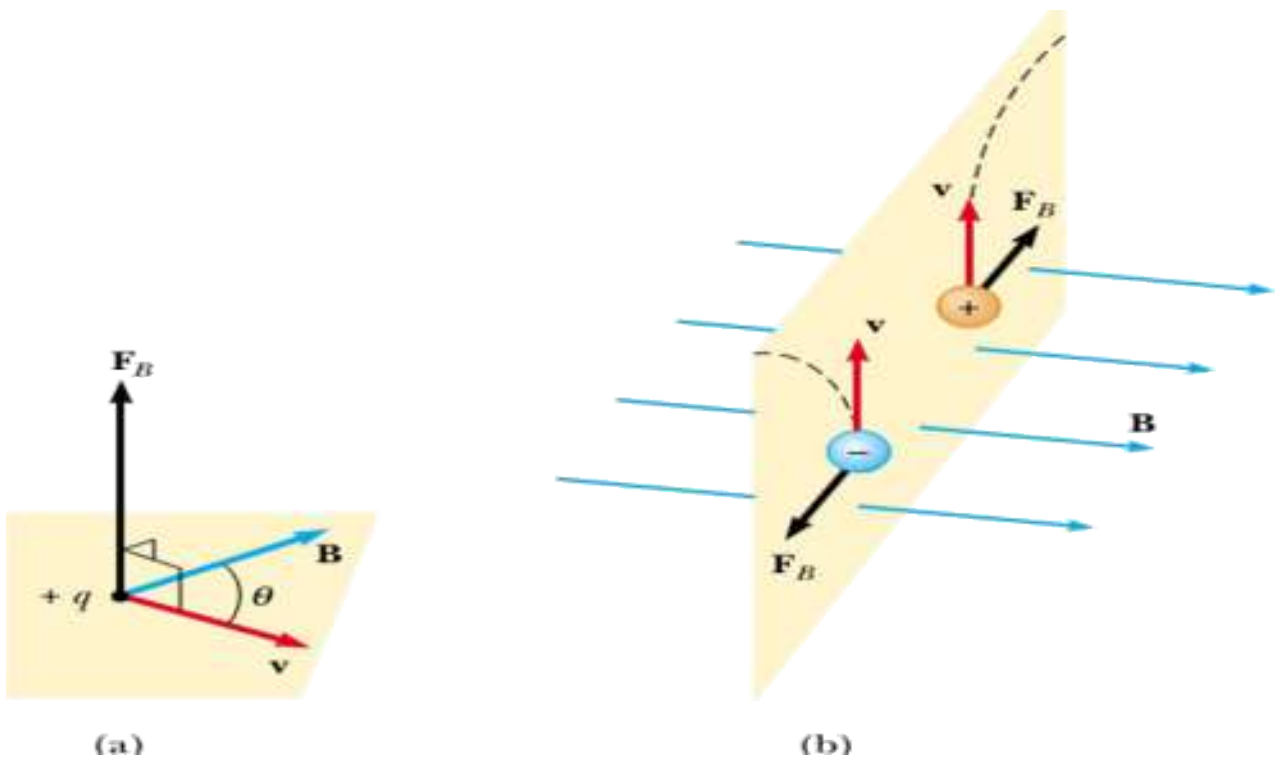


Fig.(1)

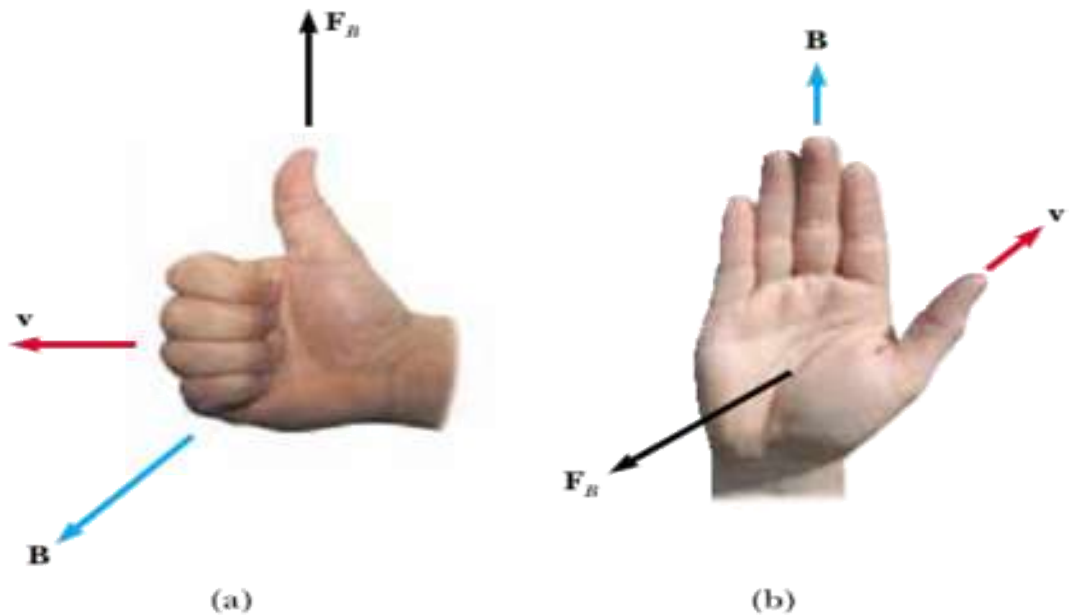


Fig.(2) Two right-hand rules for determining the direction of the magnetic force $\mathbf{F}_B = q \mathbf{v} \times \mathbf{B}$ acting on a particle with charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. (a) In this rule, the fingers point in the direction of $\mathbf{v}$, with $\mathbf{B}$ coming out of your palm, so that you can curl your fingers in the direction of $\mathbf{B}$. The direction of $\mathbf{v} \times \mathbf{B}$, and the force on a positive charge, is the direction in which the thumb points. (b) In this rule, the vector $\mathbf{v}$ is in the direction of your thumb and $\mathbf{B}$ in the direction of your fingers. The force $\mathbf{F}_B$ on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.

The magnetic force that acts on a charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \quad (2.1)$$

The direction of this magnetic force is perpendicular both to the velocity of the particle and to the magnetic field. The magnitude of this force is

$$F_B = |q|vB \sin \theta \quad (2.2)$$

where θ is the smaller angle between $\mathbf{v}$ and $\mathbf{B}$. The SI unit of $\mathbf{B}$ is the tesla(T), where $1\text{T} = 1\text{N/A.m}$.

When a charged particle moves in a magnetic field, the work done by the magnetic force on the particle is zero because the displacement is always perpendicular to the direction of the force. The magnetic field can alter the direction of the particle's velocity vector, but it cannot change its speed .

If a straight conductor of length $\mathbf{L}$ carries a current $\mathbf{I}$, the force exerted on that conductor when it is placed in a uniform magnetic field $\mathbf{B}$ is

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B} \quad (2.3)$$

where the direction of $\mathbf{L}$ is in the direction of the current and $|\mathbf{L}| = L$. If an arbitrarily shaped wire carrying a current $\mathbf{I}$ is placed in a magnetic field, the magnetic force exerted on a very small segment $d\mathbf{s}$ is

$$d\mathbf{F}_B = I d\mathbf{s} \times \mathbf{B} \quad (2.4)$$

To determine the total magnetic force on the wire, one must integrate Equation 29.4, keeping in mind that both $\mathbf{B}$ and $d\mathbf{s}$ may vary at each point. Integration gives for the force exerted on a current-carrying conductor of arbitrary shape in a uniform magnetic field.

$$\mathbf{F}_B = I \mathbf{L}' \times \mathbf{B} \quad (2.5)$$

where $\mathbf{L}'$ is a vector directed from one end of the conductor to the opposite end. Because integration of Equation 2.4 for a closed loop yields a zero result, the net magnetic force on any closed loop carrying a current in a uniform magnetic field is zero.

9. The magnetic dipole moment μ .

The magnetic dipole moment μ of a loop carrying a current I is

$$\mu = I A \quad (2.6)$$

where the area vector $\mathbf{A}$ is perpendicular to the plane of the loop and $|\mathbf{A}|$ is equal to the area of the loop. The SI unit of μ is $\text{A} \cdot \text{m}^2$.

The torque τ on a current loop placed in a uniform magnetic field $\mathbf{B}$ is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B} \quad (2.7)$$

The potential energy of the system of a magnetic dipole in a magnetic field is

$$U = -\boldsymbol{\mu} \cdot \boldsymbol{B} \quad (2.8)$$

If a charged particle moves in a uniform magnetic field so that its initial velocity is perpendicular to the field, the particle moves in a circle, the plane of which is perpendicular to the magnetic field. The radius of the circular path is

$$r = \frac{mv}{qB} \quad (2.9)$$

where m is the mass of the particle and q is its charge. The angular speed of the charged particle is

$$\omega = \frac{qB}{m} \quad (2.10)$$

CHAPTER III

HEAT

Chapter III "HEAT"

1. Temperature

The concept of temperature with how hot or cold an object feels when we touch it. We will begin a discussion of temperature and a description of one of the laws of thermodynamics (the so-called “zeroth law”). To understand the concept of temperature, it is useful to define two often-used phrases: thermal contact and thermal equilibrium. To grasp the meaning of thermal contact, imagine that two objects are placed in an insulated container such that they interact with each other but not with the environment. If the objects are at different temperatures, energy is exchanged between them, even if they are initially not in physical contact with each other.

We suppose that two objects are in thermal equilibrium with each other if they do not exchange energy when in thermal contact. The zeroth law of thermodynamics states that if objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other. Temperature is the property that determines whether an object is in thermal equilibrium with other objects. Two objects in thermal equilibrium with each other are at the same temperature.

Now let us suppose that temperatures are measured with gas thermometers containing different gases at different initial pressures. Experiments show that the thermometer readings are nearly independent of the type of gas used, as long as the gas pressure is low and the temperature is well above the point at which the gas liquefies (Fig.1). The agreement among thermometers using various gases improves as the pressure is reduced.

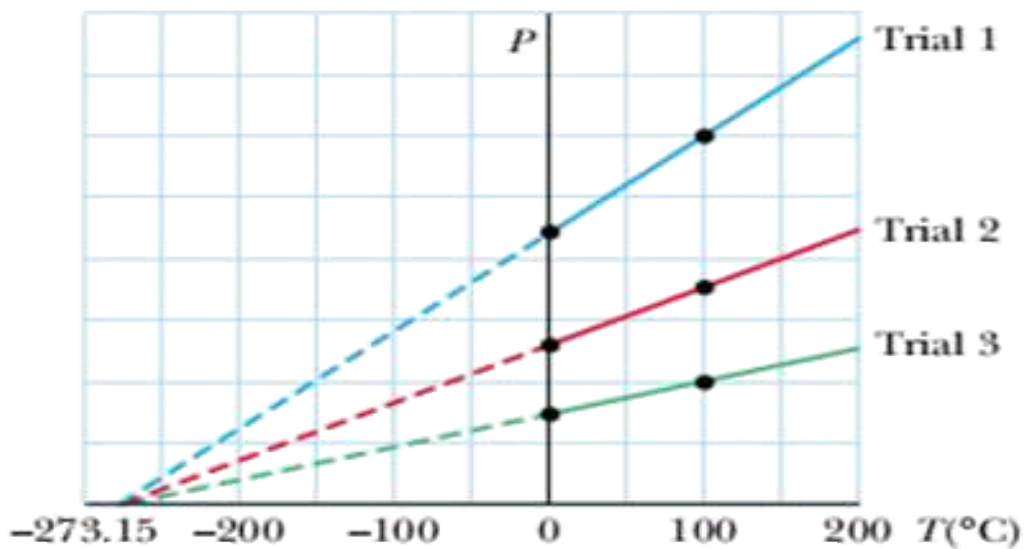


Fig.(1)

If we extend the straight lines in Figure 1 toward negative temperatures, we find a remarkable result—in every case, the pressure is zero when the temperature is 273.15°C ! This suggests some special role that this particular temperature must play. It is used as the basis for the absolute temperature scale, which sets -

273.15°C as its zero point. This temperature is often referred to as absolute zero. The size of a degree on the absolute temperature scale is chosen to be identical to the size of a degree on the Celsius scale. Thus, the conversion between these temperatures is

$$T_C = T - 273.15 \quad (1)$$

where T_C is the Celsius temperature and T is the absolute temperature. A common temperature scale in everyday use in the United States is the Fahrenheit scale. This scale sets the temperature of the ice point at 32°F and the temperature of the steam point at 212°F. The relationship between the Celsius and Fahrenheit temperature scales is

$$T_F = \frac{9}{5}T_C + 32^\circ\text{F} \quad (2)$$

We can use Equations 1 and 2 to find a relationship between changes in temperature on the Celsius, Kelvin, and Fahrenheit scales :

$$\Delta T_C = \Delta T = \frac{5}{9} \Delta T_F \quad (3)$$

The SI unit of absolute temperature is the **kelvin**, which is defined to be the fraction 1/273.16 of the temperature of the triple point of water.

2. linear and volume expansion

Suppose that an object has an initial length L_i along some direction at some temperature and that the length increases by an amount ΔL for a change in temperature ΔT . Because it is convenient to consider the fractional change in length per degree of temperature change, we define the average coefficient of linear expansion as

$$\alpha \equiv \frac{1}{L_i} \cdot \frac{\Delta L}{\Delta T}$$

$$\Delta L = \alpha L_i \Delta T ,$$

or as
$$L_f - L_i = \alpha L_i (T_f - T_i) \quad (4)$$

where the constant α is the average coefficient of linear expansion. The average coefficient of volume expansion for a solid β is approximately equal to 3α .

$$\beta = 3\alpha = \frac{1}{V_i} \cdot \frac{\Delta V}{\Delta T} \quad (5)$$

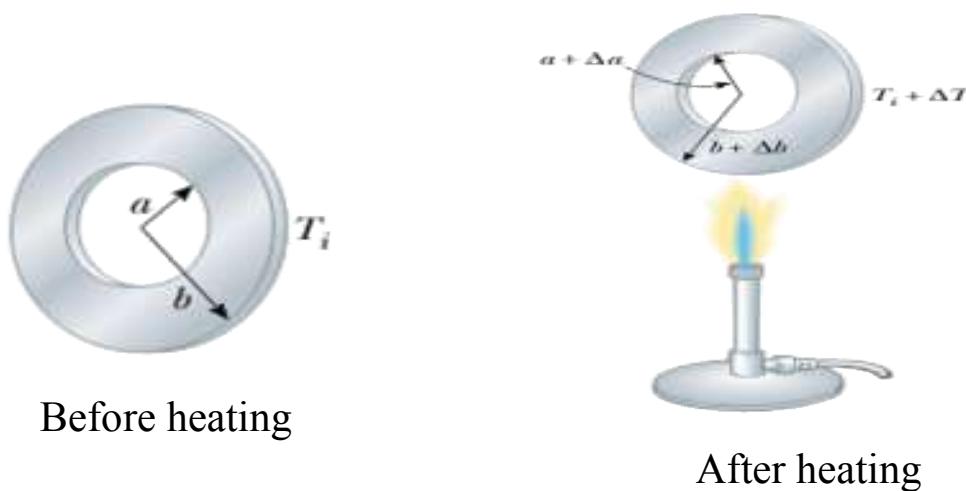


Fig.(2) Thermal expansion of a homogeneous metal washer.
As the washer is heated, all dimensions increase.

The average coefficient of linear expansion for various materials is listed, shown in the table 1. . Note that for these materials α is positive, indicating an increase in length with increasing temperature.

Table 1.

Average Expansion Coefficients for Some Materials Near Room Temperature			
Material	Average Linear Expansion Coefficient (α)($^{\circ}\text{C}$) ⁻¹	Material	Average Volume Expansion Coefficient (β)($^{\circ}\text{C}$) ⁻¹
Aluminum	24×10^{-6}	Alcohol, ethyl	1.12×10^{-4}
Brass and bronze	19×10^{-6}	Benzene	1.24×10^{-4}
Copper	17×10^{-6}	Acetone	1.5×10^{-4}
Glass (ordinary)	9×10^{-6}	Glycerin	4.85×10^{-4}
Glass (Pyrex)	3.2×10^{-6}	Mercury	1.82×10^{-4}
Lead	29×10^{-6}	Turpentine	9.0×10^{-4}
Steel	11×10^{-6}	Gasoline	9.6×10^{-4}
Invar (Ni-Fe alloy)	0.9×10^{-6}	Air ^a at 0°C	3.67×10^{-3}
Concrete	12×10^{-6}	Helium ^a	3.665×10^{-3}

We can summarize these results in a statement known as the zeroth law of thermodynamics (the law of equilibrium) : " If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other." For a gas, it is useful to know how the quantities volume V, pressure P, and temperature T are related for a sample of gas of mass m. In

general, the equation that interrelates these quantities, called the equation of state, is very complicated. However, if the gas is maintained at a very low pressure (or low density), the equation of state is quite simple and can be found experimentally. Such a low-density gas is commonly referred to as an ideal gas. One mole of any substance is that amount of the substance that contains Avogadro's number $N_A = 6.022 \times 10^{23}$ of constituent particles (atoms or molecules). The number of moles n of a substance is related to its mass m through the expression $n = m/M$ where M is the molar mass of the substance.

Now suppose that an ideal gas is confined to a cylindrical container whose volume can be varied by means of a movable piston, as in Figure 3. If we assume that the cylinder does not leak, the mass (or the number of moles) of the gas remains constant. For such a system, experiments provide the following information. First, when the gas is kept at a constant temperature, its pressure is inversely proportional to its volume (Boyle's law). Second, when the pressure of the gas is kept constant, its volume is directly proportional to its temperature (the law of Charles and Gay-Lussac).



Fig.(3)

These observations are summarized by the equation of state for an ideal gas:

$$PV = nRT \quad (6)$$

where n equals the number of moles of the gas, V is its volume, R is the universal gas constant (8.314J/mol.K or $0.08314\text{ L.atm/mol.K}$), and T is the absolute temperature. A real gas behaves approximately as an ideal gas if it has a low density.

3. Heat and Internal Energy

It is important that we make a major distinction between internal energy and heat.

Internal energy is all of a system's energy that is associated with the system's microscopic components. Internal energy includes kinetic energy of random translation, rotation, and vibration of molecules, potential energy within molecules, and potential energy between molecules.

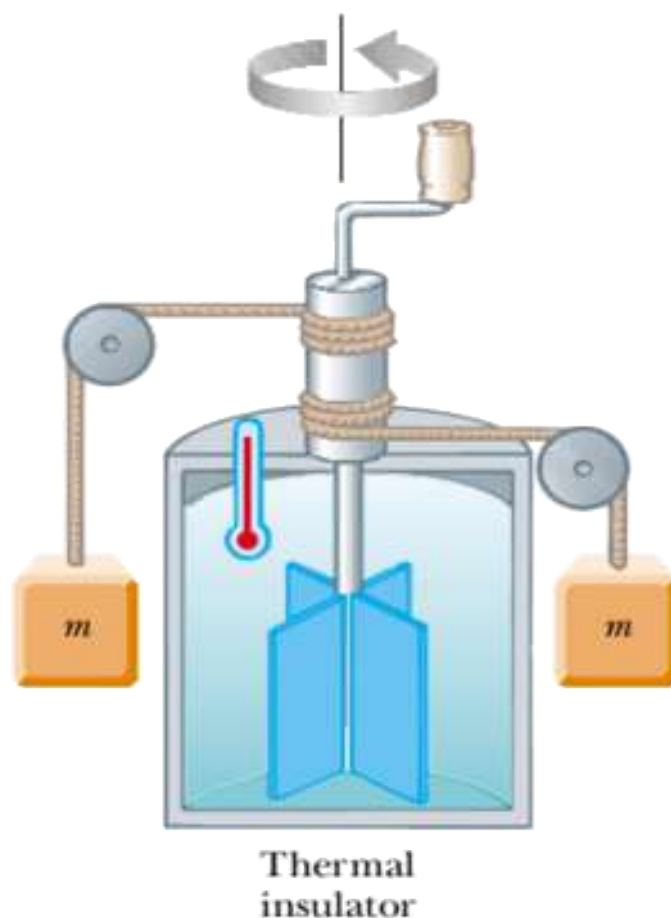
Heat is the transfer of energy across the boundary of a system resulting from a temperature difference between the system and its surroundings. We use the symbol Q for the amount of energy transferred by this process.

Units of Heat : *The calorie* is the amount of energy necessary to raise the temperature of 1g of water from 14.5°C to 15.5°C. The mechanical equivalent of heat is $1\text{ cal} = 4.186\text{ J}$.

The heat capacity C of any sample is the amount of energy needed to raise the temperature of the sample by 1°C.

4. The Mechanical Equivalent of Heat:

A schematic diagram of Joule's most famous experiment for determining is shown in Figure 3. .



Fig,(3)

The system of interest is the water in a thermally insulated container. Work is done on the water by a rotating paddle wheel, which is driven by heavy blocks falling at a constant speed. The temperature of the stirred water increases due to the friction between it and the paddles. If the energy lost in the bearings and through the walls is neglected, then the loss in potential energy associated with the blocks equals the work done by the paddle wheel on the water. If the two blocks fall through a distance h , the loss in potential energy is $2mgh$, where m is the mass of one block; this energy causes the temperature of the water to increase. By varying the conditions of the experiment, Joule found that the loss in mechanical energy $2mgh$ is proportional to the increase in water temperature ΔT . The proportionality constant was found to be approximately $4.18\text{J/g}\cdot^\circ\text{C}$. Hence, 4.18J of mechanical energy raises the temperature of 1 g of water by 1°C . More precise measurements taken later demonstrated the proportionality to be $4.186\text{J/g}\cdot^\circ\text{C}$ when the temperature of the water was raised from 14.5°C to 15.5°C . We adopt this "15-degree calorie" value : $1\text{cal} = 4.186\text{ J}$. This equality is known as the mechanical equivalent of heat.

5. Specific Heat and Heat Capacity, and Calorimetry

If the system consists of a sample of a sub-stance, we find that the quantity of energy required to raise the temperature of a given mass of the substance by some amount varies from one substance to another. For example, the quantity of energy required to raise the temperature of 1kg of water by 1°C is 4186J, but the quantity of energy required to raise the temperature of 1kg of copper by 1°C is only 387J. In the discussion that follows, we shall use heat as our example of energy transfer, but keep in mind that we could change the temperature of our system by means of any method of energy transfer.

The heat capacity C of a particular sample of a substance is defined as the amount of energy needed to raise the temperature of that sample by 1°C. From this definition we see that if energy Q produces a change ΔT in the temperature of a sample, then

$$Q = C\Delta T \quad (7)$$

The specific heat c of a substance is the heat capacity per unit mass. Thus, if energy Q transfers to a sample of a substance with mass m and the temperature of the sample changes by ΔT , then the specific heat of the substance is

$$c \equiv \frac{Q}{m \Delta T} \quad (8)$$

The energy Q required to change the temperature of a mass m of a substance by an amount ΔT is

$$Q = mc \Delta T \quad (9)$$

where c is the specific heat of the substance. Table 20.1 lists representative specific heats.

For example, the energy required to raise the temperature of 0.5kg of water by 3. °C is $(0.5\text{kg}) (4186\text{J/kg}\cdot^{\circ}\text{C})(3^{\circ}\text{C}) = 6.28 \times 10^3 \text{ J}$. Note that when the temperature increases, Q and ΔT are taken to be positive, and energy transfers into the system. When the temperature decreases, Q and ΔT are negative, and energy transfers out of the system. For solids and liquids, the difference between the two values is usually no greater than a few percent and is often neglected. Most of the values given in Table 2 were measured at atmospheric pressure and room temperature. The specific heats for gases measured at constant pressure are quite different from values measured at constant volume.

6. Latent Heat

If a quantity Q of energy transfer is required to change the phase of a mass m of a substance, the ratio $L \equiv Q/m$ characterizes an important thermal property of that substance. Because this added or removed energy does not result in a temperature change, the quantity L is called the latent heat (literally, the “hidden” heat) of the substance. The value of L for a substance depends on the nature of the phase change, as well as on the properties of the substance. From the definition of latent heat, and again choosing heat as our energy transfer mechanism, we find that the energy required to

change the phase of a given mass m of a pure substance is

$$Q = \pm mL \quad (8)$$

Table 2.

Specific Heats of Some Substances at 25°C and Atmospheric Pressure		
Substance	Specific heat c	
	J/kg · °C	cal/g · °C
<i>Elemental solids</i>		
Aluminum	900	0.215
Beryllium	1 830	0.436
Cadmium	230	0.055
Copper	387	0.092 4
Germanium	322	0.077
Gold	129	0.030 8
Iron	448	0.107
Lead	128	0.030 5
Silicon	703	0.168
Silver	234	0.056
<i>Other solids</i>		
Brass	380	0.092
Glass	837	0.200
Ice (− 5°C)	2 090	0.50
Marble	860	0.21
Wood	1 700	0.41
<i>Liquids</i>		
Alcohol (ethyl)	2 400	0.58
Mercury	140	0.033
Water (15°C)	4 186	1.00
<i>Gas</i>		
Steam (100°C)	2 010	0.48

where L is the latent heat of the substance and depends on the nature of the phase change and the properties of the substance. The positive sign is used if energy is entering the system, and the negative sign is used if energy is leaving. The latent heats of various substances vary considerably, as data in Table 3. show.

Part A. On this portion of the curve, the temperature of the ice changes from 30.0°C to 0.0°C . Because the specific heat of ice is $2090\text{J/kg}\cdot^{\circ}\text{C}$, we can calculate the amount of energy added as

$$\begin{aligned} Q &= m_i c_i \Delta T = (1.0 \times 10^{-3}\text{kg})(2.09\text{J/kg}\cdot^{\circ}\text{C})(30.0^{\circ}\text{C}) \\ &= 62.7\text{J} \end{aligned}$$

Table 3.

Latent Heats of Fusion and Vaporization				
Substance	Melting Point ($^{\circ}\text{C}$)	Latent Heat of Fusion (J/kg)	Boiling Point ($^{\circ}\text{C}$)	Latent Heat of Vaporization (J/kg)
Helium	-269.65	5.23×10^3	-268.93	2.09×10^4
Nitrogen	-209.97	2.55×10^4	-195.81	2.01×10^5
Oxygen	-218.79	1.38×10^4	-182.97	2.13×10^5
Ethyl alcohol	-114	1.04×10^5	78	8.54×10^5
Water	0.00	3.33×10^5	100.00	2.26×10^6
Sulfur	119	3.81×10^4	444.60	3.26×10^5
Lead	327.3	2.45×10^4	1750	8.70×10^5
Aluminum	660	3.97×10^5	2450	1.14×10^7
Silver	960.80	8.82×10^4	2193	2.33×10^6
Gold	1063.00	6.44×10^4	2660	1.58×10^6
Copper	1083	1.34×10^5	1187	5.06×10^6

Part B. When the temperature of the ice reaches 0.0°C, the ice–water mixture re-mains at this temperature even though energy is being added until all the ice melts. The energy required to melt 1.00g of ice at 0.0°C is,

$$Q = m_i L_f = (1.0 \times 10^{-3} \text{ kg})(3.33 \times 10^5 \text{ J/kg}) = 333 \text{ J}$$

Thus, we have moved to the $(396 \text{ J} - 62.7 \text{ J} = 333 \text{ J})$ mark on the energy axis in Figure4.

Part C. Between 0.0°C and 100.0°C, nothing surprising happens. No phase change occurs, and so all energy added to the water is used to increase its temperature. The amount of energy necessary to increase the temperature from 0.0°C to 100.0°C is

$$\begin{aligned} Q &= m_w c_w \Delta T \\ &= (1.0 \times 10^{-3} \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot ^\circ\text{C})(100.0 ^\circ\text{C}) = 419 \text{ J} \end{aligned}$$

Part D. At 100.0°C, another phase change occurs as the water changes from water at 100.0°C to steam at 100.0°C. Similar to the ice–water mixture in part B, the water–steam mixture remains at 100.0°C—even though energy is being added—until all of the liquid has been converted to steam. The energy required to convert 1.00g of water to steam at 100.0°C is

$$\begin{aligned} Q &= m_w \times L_v = (1.0 \times 10^{-3} \text{ kg})(2.26 \times 10^6 \text{ J/kg}) \\ &= 2.26 \times 10^3 \text{ J} \end{aligned}$$

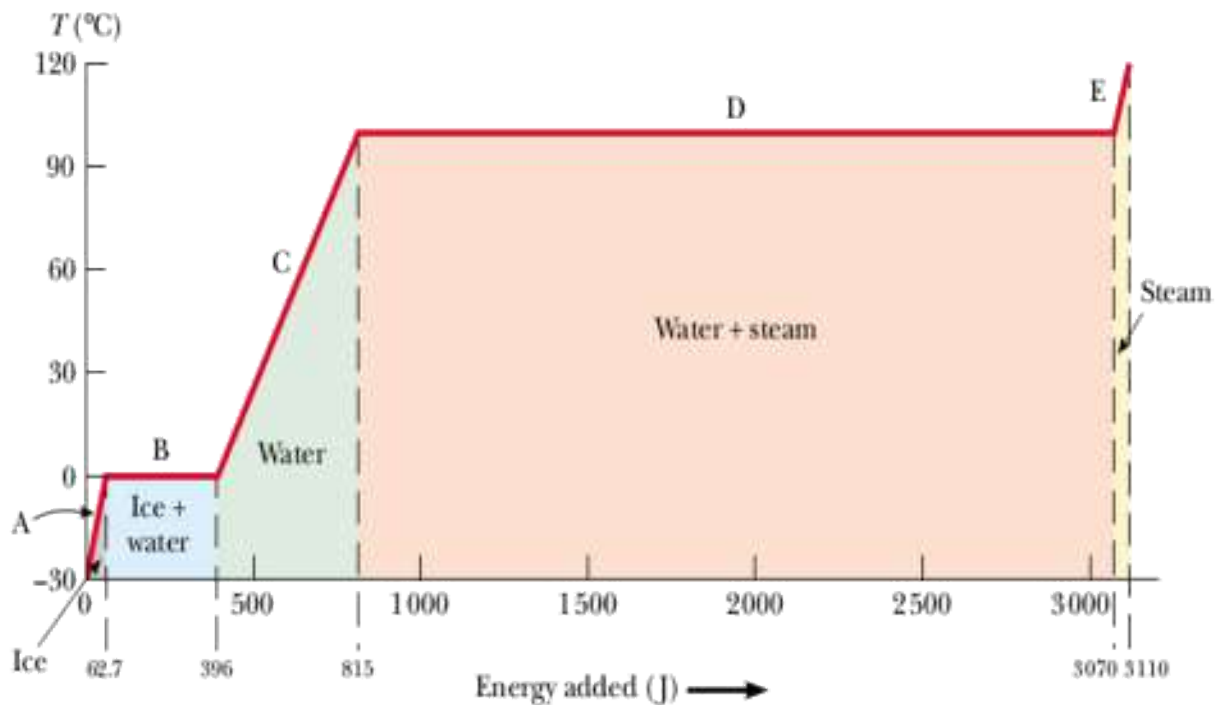


Fig.(4)

Part E. On this portion of the curve, as in parts A and C, no phase change occurs; thus, all energy added is used to increase the temperature of the steam. The energy that must be added to raise the temperature of the steam from 100.0°C to 120.0°C is

$$Q = m_s C_s \Delta T$$

$$= (1.0 \times 10^{-3} \text{ kg})(2.01 \times 10^3 \text{ J/kg} \cdot ^\circ\text{C})(20.0 ^\circ\text{C}) = 40.2 \text{ J}$$

The total amount of energy that must be added to change 1g of ice at - 30.0°C to steam at 120.0°C is the sum of the results from all five parts of the curve, which is $3.11 \times 10^3 \text{ J}$. Conversely, to cool 1g of steam at 120.0°C to ice at - 30.0°C, we must remove $3.11 \times 10^3 \text{ J}$ of energy.

7. Work and Heat in Thermodynamic Processes

We study another important transfer variable for thermodynamic systems—work. The work performed on particles can be investigated to be done on a deformable system—a gas. Consider a gas contained in a cylinder fitted with a movable piston (Fig.5). At equilibrium, the gas occupies a volume V and exerts a uniform pressure P on the cylinder's walls and on the piston .

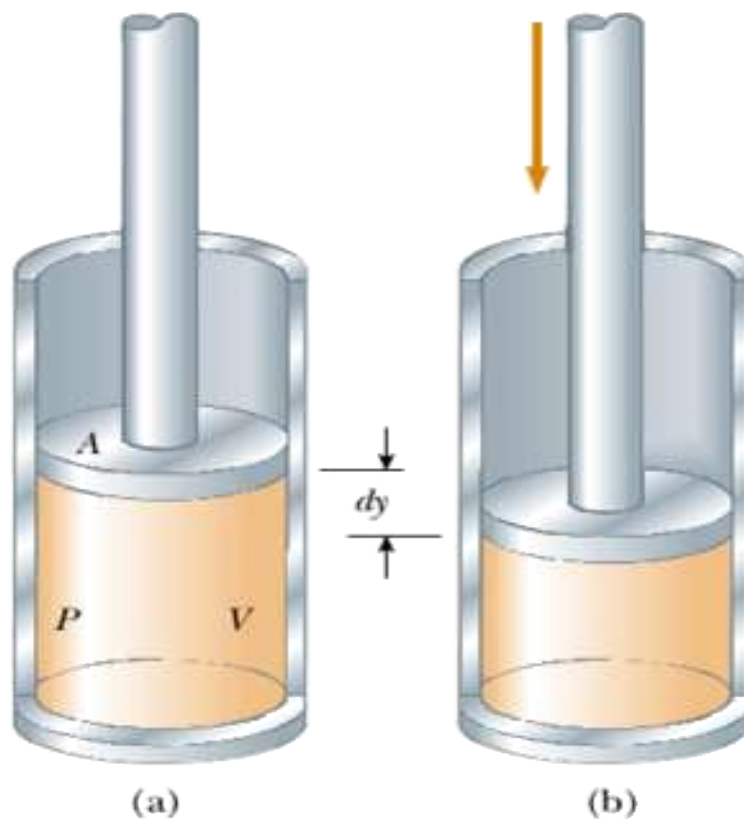


Fig.(5) Work is done on a gas contained in a cylinder at a pressure P as the piston is pushed downward so that the gas is compressed.

The work done on a gas as its volume changes from some initial value V_i to some final value V_f

is

$$W = - \int_{V_i}^{V_f} P dV \quad (9)$$

where P is the pressure, which may vary during the process. In order to evaluate W , the process must be fully specified—that is, P and V must be known during each step. In other words, the work done depends on the path taken between the initial and final states. In general, the pressure is not constant during a process followed by a gas, but depends on the volume and temperature. If the pressure and volume are known at each step of the process, the state of the gas at each step can be plotted on a graph called a **PV diagram**, as in Figure 6. This type of diagram allows us to visualize a process through which a gas is progressing. The curve on a **PV diagram** is called the *path* taken between the initial and final states. Note that the integral in Equation 9 is equal to the area under a curve on a **PV diagram**. Thus, we can identify an important use for **PV diagrams**: "*The work done on a gas in a quasi-static process that takes the gas from an initial state to a final state is the negative of the area under the curve on a PV diagram, evaluated between the initial and final states*".

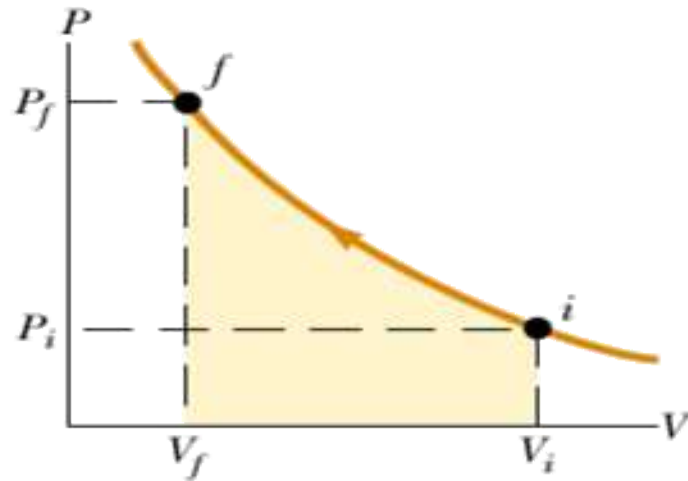


Fig.(6) A gas is compressed quasi-statically (slowly) from state i to state f . The work done on the gas equals the negative of the area under the PV curve.

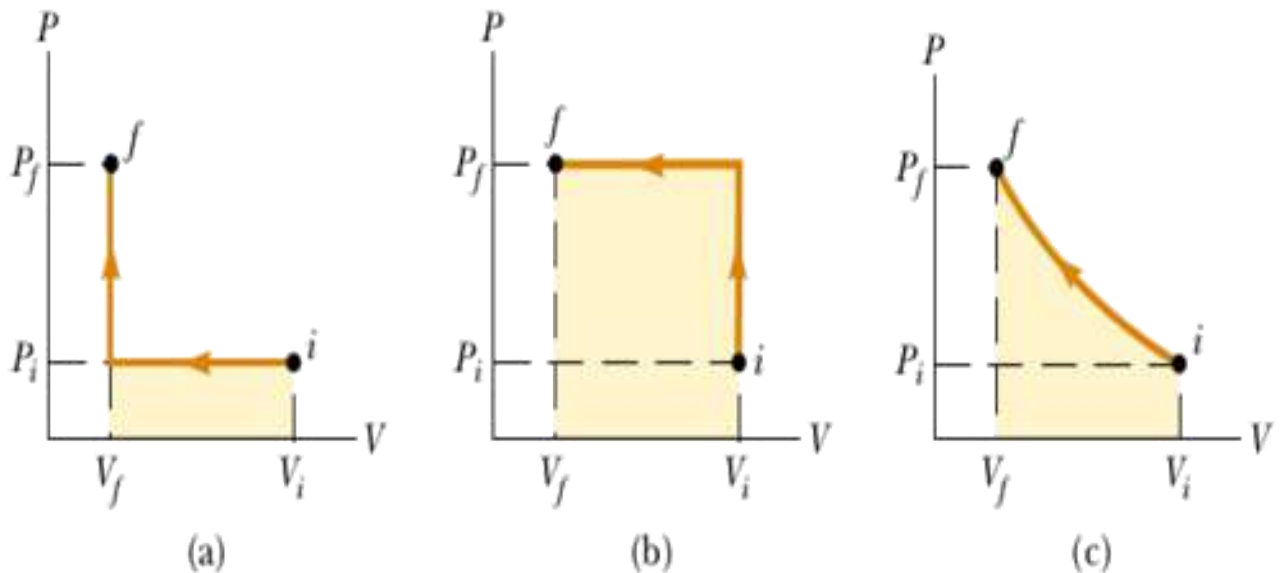


Fig.(7) The work done on a gas as it is taken from an initial state to a final state depends on the path between these states .

As Figure 6 suggests, for our process of compressing a gas in the cylinder, the work done depends on the particular path taken between the initial and final states. To illustrate this important point, consider several different paths connecting i and f (Fig. 7).

In the process depicted in Figure 7a, the volume of the gas is first reduced from V_i to V_f at constant pressure P_i and the pressure of the gas then increases from P_i to P_f by heating at constant volume V_f . The work done on the gas along this path is $-P_i(V_f - V_i)$. In Figure 7b, the pressure of the gas is increased from P_i to P_f at constant volume V_i and then the volume of the gas is reduced from V_i to V_f at constant pressure P_f . The work done on the gas is $-P_f(V_f - V_i)$ (which is greater than that for the process described in Figure 7a. It is greater because the piston is moved through the same displacement by a larger force than for the situation in Figure 7a. Finally, for the process described in Figure 7c, where both P and V change continuously, the work done on the gas has some value intermediate between the values obtained in the first two processes. To evaluate the work in this case, the function $P(V)$ must be known, so that we can evaluate the integral in Equation 9.

The energy transfer Q into or out of a system by heat also depends on the process. (Explain?)

The first law of thermodynamics states that when a system undergoes a change from one state to another, the change in its internal energy is

$$\Delta E_{int} = Q + W \quad (10)$$

where Q is the energy transferred into the system by heat and W is the work done on the system. Although Q and W both depend on the path taken from the initial state to the final state, the quantity ΔE_{int} is path-independent

- In a *cyclic process* (one that originates and terminates at the same state), $\Delta E_{int} = 0$ and, therefore, $Q = -W$. That is, the energy transferred into the system by heat equals the negative of the work done on the system during the process.
- In an *adiabatic process*, no energy is transferred by heat between the system and its surroundings ($Q = 0$). In this case, the first law gives $\Delta E_{int} = W$. That is, the internal energy changes as a consequence of work being done on the system. In the adiabatic free expansion of a gas $Q = 0$ and $W = 0$, and so $\Delta E_{int} = 0$. That is, the internal energy of the gas does not change in such a process.
- An *isobaric process* is one that occurs at constant pressure. The work done on a gas in such a process is $W = -P(V_f - V_i)$.
- An *isovolumetric process* is one that occurs at constant volume. No work is done in such a process, so $\Delta E_{int} = Q$.

- **An isothermal process** is one that occurs at constant temperature. The work done on an ideal gas during an isothermal process is

$$W = nRT \ln \left(\frac{V_i}{V_f} \right) \quad (11)$$

Numerically, this work W equals the negative of the shaded area under the PV curve shown in Figure 6 . Because the gas expands, $V_f > V_i$ and the value for the work done on the gas is negative, as we expect. If the gas is compressed, then $V_f < V_i$ and the work done on the gas is positive.

8. Energy Transfer Mechanisms

The **process of energy transfer** by heat can also be called ***conduction or thermal conduction***. In this process, the transfer can be represented on an atomic scale as an exchange of kinetic energy between microscopic particles—molecules, atoms, and free electrons—in which less-energetic particles gain energy in collisions with more energetic particles. The rate of thermal conduction depends on the properties of the substance being heated.

If there is only a difference in temperature between two parts of the conducting medium, conduction occurs . Consider a slab of

material of thickness Δx and cross-sectional area A . One face of the slab is at a temperature T_c and the other face is at a temperature $T_h > T_c$ (Fig. 8). Experimentally, it is found that the energy Q transfers in a time interval Δt from the hotter face to the colder one. The rate $P = Q/\Delta t$ at which this energy transfer occurs is found to be proportional to the cross-sectional area and the temperature difference $\Delta T = T_h - T_c$, and inversely proportional to the thickness :

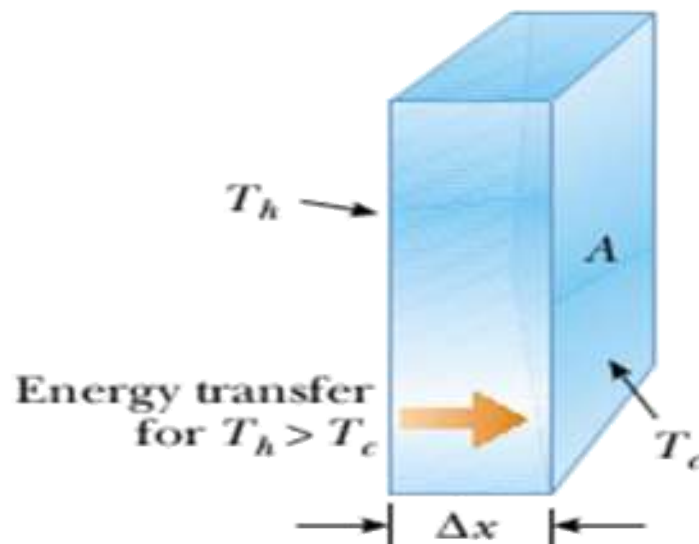


Fig.(8) Energy transfer through a conducting slab with a cross-sectional area A and a thickness Δx . The opposite faces are at different temperatures T_c and T_h .

$$P = \frac{Q}{\Delta t} \propto A \frac{\Delta T}{\Delta x} \quad (12)$$

Note that P has units of watts when Q is in joules and Δt is in seconds. This is not surprising because P is *power*—the rate of

energy transfer by heat. For a slab of infinitesimal thickness dx and temperature difference dT , we can write the **law of thermal conduction** " *The rate of energy transfer by conduction through a slab of area A* " as

$$P = kA \left| \frac{dT}{dx} \right| \quad (13)$$

where k is the thermal conductivity of the material from which the slab is made and $|dT/dx|$ is the temperature gradient. This equation can be used in many situations in which the rate of transfer of energy through materials is important. In convection, a warm substance transfers energy from one location to another.

Suppose that a long, uniform rod of length L is thermally insulated so that energy cannot escape by heat from its surface except at the ends, as shown in Figure 20.11.

One end is in thermal contact with an energy reservoir at temperature T_c , and the other end is in thermal contact with a reservoir at temperature $T_h > T_c$. When a steady state has been reached, the temperature at each point along the rod is constant in time. In this case if we assume that k is not a function of temperature, the temperature gradient is the same everywhere along the rod and is

$$\left| \frac{dT}{dx} \right| = \frac{T_h - T_c}{L} \quad (14)$$

Thus the rate of energy transfer by conduction through the rod is

$$P = kA \left(\frac{T_h - T_c}{L} \right) \quad (15)$$

Substances that are good thermal conductors have large thermal conductivity values, whereas good thermal insulators have low thermal conductivity values. Table 20.3 lists thermal conductivities for various substances . For a compound slab containing several materials of thicknesses $L_1, L_2, \dots$ and thermal conductivities $k_1, k_2, \dots$, the rate of energy transfer through the slab at steady state is

$$P = \frac{A(T_h - T_c)}{\sum_i (L_i / k_i)} \quad (16)$$

where T_c and T_h are the temperatures of the outer surfaces (which are held constant) and the summation is over all slabs .

9. Convection and Radiation.

At one time or another, you probably have warmed your hands by holding them over an open flame. In this situation, the air directly above the flame is heated and expands. As a result, the density of this air decreases and the air rises. This hot air warms your hands as it flows by. *Energy transferred by the movement of a warm substance is said to have been transferred by **convection**.* When the movement results from differences in density, as with air around a fire, it is referred to as **natural convection**. Air flow at a

beach is an example of natural convection, as is the mixing that occurs as surface water in a lake cools and sinks. When the heated substance is forced to move by a fan or pump, as in some hot-air and hot-water heating systems, the process is called ***forced convection*** .

All objects radiate energy continuously in the form of electromagnetic waves produced by thermal vibrations of the molecules. *The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature.* This is known as ***Stefan's law*** and is expressed in equation form as

$$P_{net} = \sigma A e T^4 \quad (13)$$

where P is the power in watts radiated from the surface of the object, σ is a constant equal to $5.6696 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$, A is the surface area of the object in square meters, e is the emissivity, and T is the surface temperature in kelvins. The value of e can vary between zero and unity, depending on the properties of the surface of the object. The emissivity e is equal to the absorptivity, which is the fraction of the incoming radiation that the surface absorbs. It is known *an object that is hotter than its surroundings radiates more energy than it absorbs, whereas an object that is cooler than its surroundings absorbs more energy than it radiates* .If an object is at a temperature T and its surroundings are at an average

temperature T_0 , then the net rate of energy gained or lost by the object as a result of radiation is

$$P_{net} = \sigma A e (T^4 - T_o^4) \quad (14)$$

When an object is in *equilibrium* with its surroundings, it radiates and absorbs energy at the *same rate*, and its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs, and its temperature decreases.

An ideal absorber is defined as *an object that absorbs all the energy incident on it*, and for such an object, $e = 1$. An object for which $e = 1$ is often referred to as a black body. It can be investigated by experimental and theoretical approaches to radiation from a black body. *An ideal absorber* is also an ideal radiator of energy. In contrast, an object for which $e = 0$ absorbs none of the energy incident on it. Such an object reflects all the incident energy, and thus is *an ideal reflector*.

CHAPTER IV

PROPERTIES

OF

MATTER

1. Elasticity

Elasticity is the property of an object or material which causes it to be restored to its original shape after distortion. It is said to be more elastic if it restores itself more precisely to its original configuration. A spring is an example of an elastic object - when stretched, it exerts a restoring force which tends to bring it back to its original length. This restoring force is generally proportional to the amount of stretch, as described by Hooke's Law. For wires or columns, the elasticity is generally described in terms of the amount of deformation (strain) resulting from a given stress (Young's modulus). Bulk elastic properties of materials describe the response of the materials to changes in pressure (Bulk's modulus) .

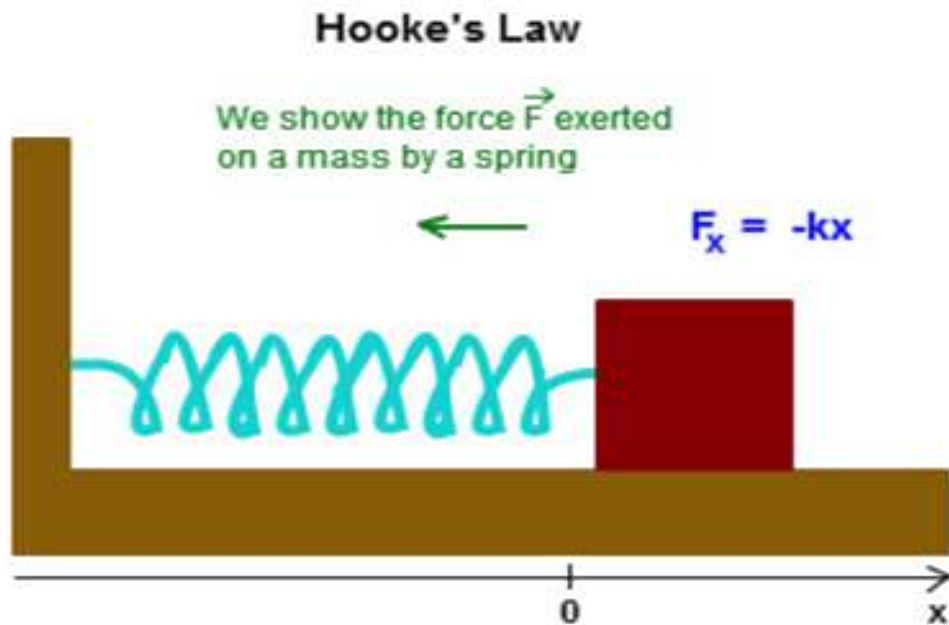
1.1. Elastic Properties of Solids

One of the properties of elasticity is that it takes about twice as much force to stretch a spring twice as far. That linear dependence of displacement upon stretching force is called Hooke's law.

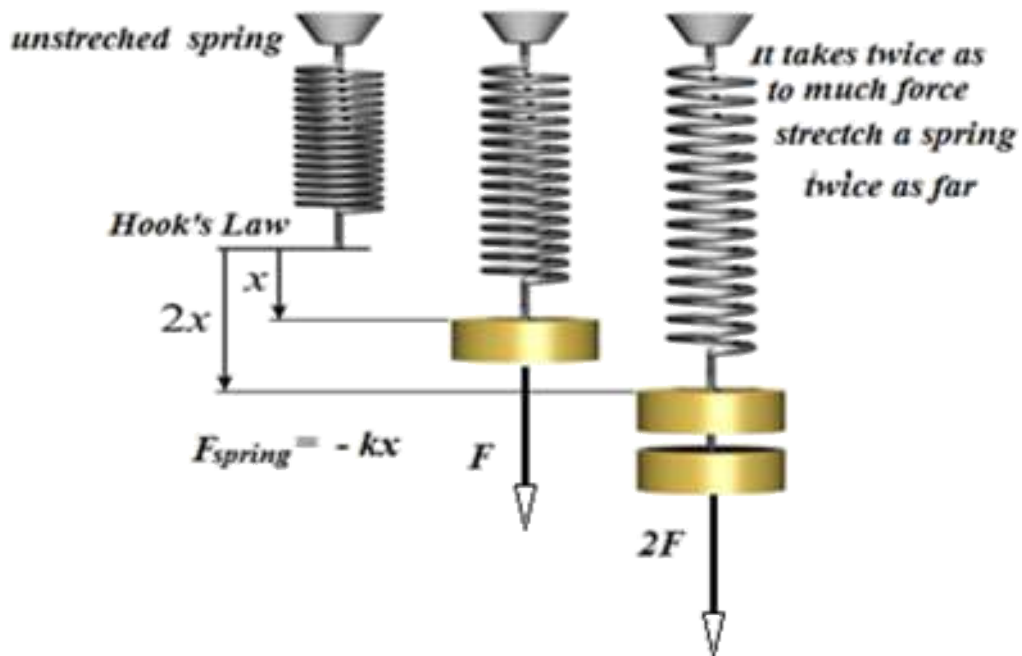
1.1.a For linear springs "Hook's Law"

Consider a simple helical spring that has one end attached to some

fixed object, while the free end is being pulled by a force whose magnitude is F .



(a)



(b)

Fig.(1a,b)

Suppose that the spring has reached a state of equilibrium, where its length is not changing anymore.

Let x be the amount by which the free end of the spring was displaced from its "relaxed" position (when it is not being stretched) horizontally or vertically as shown in Fig.(1 a,b).

Hooke's law states that

$$F = k x \quad (1)$$

or, equivalently, $x = F/k$, where k is a positive real number, characteristic of the spring. Moreover, the same formula holds when the spring is compressed, with F and x both negative in that case.

According to this formula, the graph of the applied force F as a function of the displacement x will be a straight line passing through the origin, whose slope is k .

Hooke's law for a spring is often stated under the convention that F is the restoring (reaction) force exerted by the spring on whatever is pulling its free end. In that case the equation becomes

$$F = - kx \quad (2)$$

since the direction of the restoring force is opposite to that of the displacement.

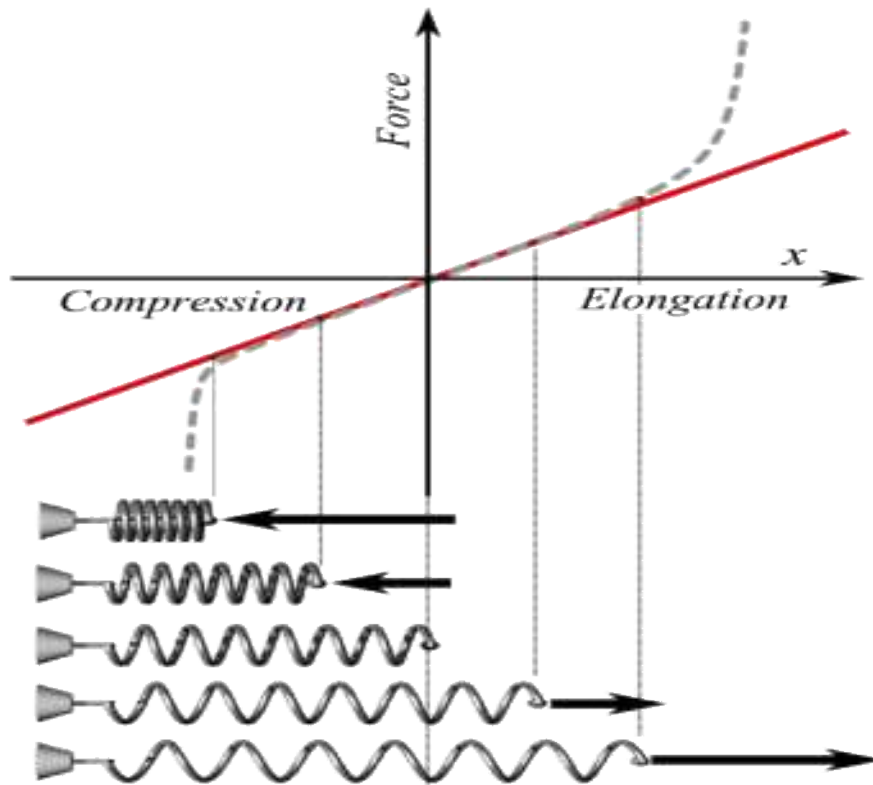


Fig.(2) Plot of applied force F vs. elongation x for a helical spring according to Hooke's law (solid line) and what the actual plot might look like (dashed line). At bottom, pictures of spring states corresponding to some points of the plot; the middle one is in the relaxed state (no force applied).

1.1.b Spring energy

The potential energy stored in a spring is given by

$$U_{el}(x) = \frac{1}{2} kx^2 \quad (3)$$

which comes from adding up the energy it takes to incrementally compress the spring. That is, the integral of force over displacement. Since the external force has the same general

direction as the displacement, the potential energy of a spring is always non-negative. This potential $U_{el}(\mathbf{x})$ can be visualized as a parabola on the $U\text{-}\mathbf{x}$ plane such that Eq.(3) . As the spring is stretched in the positive $\mathbf{x}$ -direction, the potential energy increases parabolically (the same thing happens as the spring is compressed). Since the change in potential energy changes at a constant rate:

$$\frac{d^2}{dx^2} U_{el} = k \quad (4)$$

Note that the change in the change in U is constant even when the displacement and acceleration are zero.

1.2. Elastic Modulus

We can describe the elastic properties of a substance using the concepts of *stress* and *strain*. *Stress* is a quantity proportional to the force producing a deformation; strain is a measure of the degree of deformation. *Strain* is proportional to *stress*, and the constant of proportionality is the *elastic modulus*:

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}} \quad (5)$$

Three common types of deformation are represented by

1. the resistance of a solid to elongation under a load, characterized by *Young's modulus* Y ;
2. the resistance of a solid to the motion of internal planes sliding past each other, characterized by the *shear modulus* S ; and
3. the resistance of a solid or fluid to a volume change, characterized by the *bulk modulus* B .

1.2.a Young's Modulus: Elasticity in Length

Consider a long bar of cross-sectional area A and initial length L_i that is clamped at one end, as in Figure 1. When an external force is applied perpendicular to the cross section, internal forces in the bar resist distortion (“**stretching**”), but the bar reaches an equilibrium situation in which its final length L_f is greater than L_i and in which the external force is exactly balanced by **internal forces**. In such a situation, the bar is said to be stressed.

We define the **tensile stress** as the ratio of the magnitude of the external force F to the cross-sectional area A .

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i} \quad (6)$$

The tensile strain in this case is defined as the ratio of the change in length ΔL to the original length L_i . We define *Young's modulus* by a combination of these two ratios:

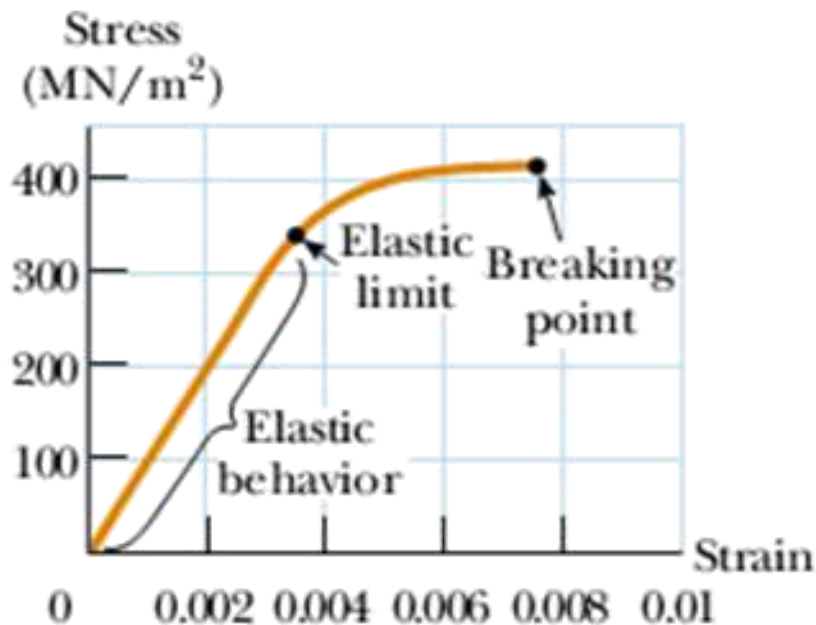


Fig.(3) Stress-versus-strain curve for an elastic solid.

Table(1)

Typical Values for Elastic Moduli			
Substance	Young's Modulus (N/m ²)	Shear Modulus (N/m ²)	Bulk Modulus (N/m ²)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5\text{--}7.8 \times 10^{10}$	$2.6\text{--}3.2 \times 10^{10}$	$5.0\text{--}5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression. Note that because strain is a dimensionless quantity, Y has units of force per unit area. Typical values are given in Table 1. Experiments show (a) that for a fixed applied force, the change in length is proportional to the original length and (b) that the force necessary to produce a given strain is proportional to the cross-sectional area. Both of these observations are in accord with Equation 6.

For relatively small stresses, the bar will return to its initial length when the force is removed. The elastic limit of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress, as seen in Figure 3.

Initially, a stress-versus-strain curve is a straight line. As the stress increases, however, the curve is no longer a straight line.

When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. As the stress is increased even further, the material ultimately breaks .

1.2.b Shear Modulus: Elasticity of Shape

Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force (Fig. 4). The stress in this case is called a **shear stress**.

If the object is originally a rectangular block, a **shear stress** results in a shape whose cross section is a parallelogram.

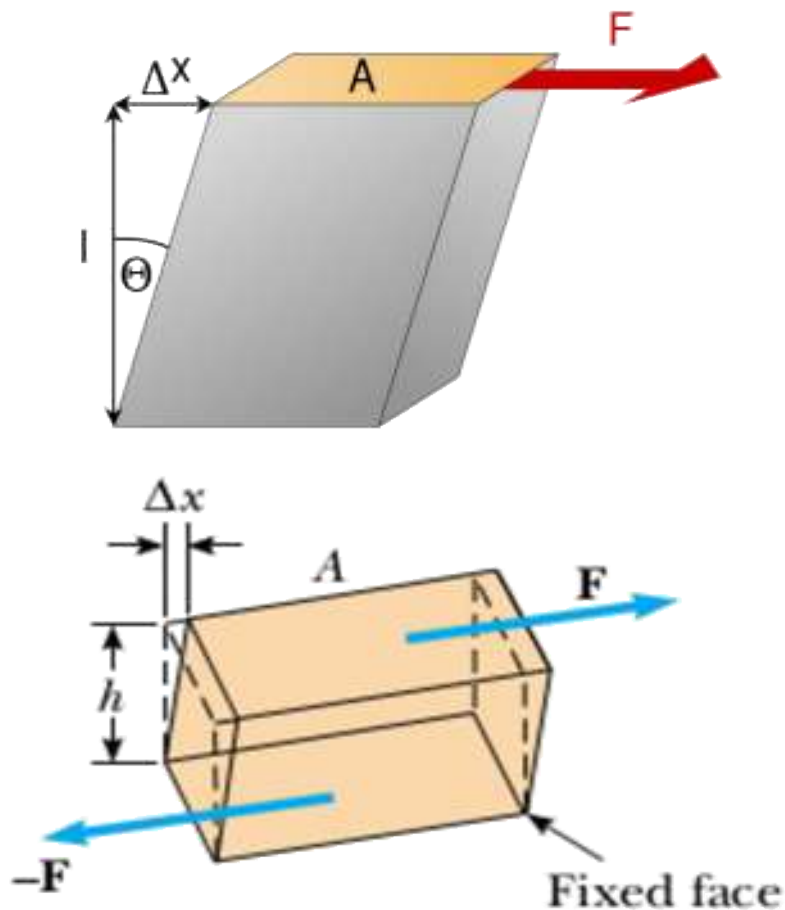


Fig.(4) A shear deformation in which a rectangular block is distorted by two forces of equal magnitude but opposite directions applied to two parallel faces.

We define the shear stress as F/A , the ratio of the tangential force to the area A of the face being sheared. The shear strain is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object. In terms of these quantities, the shear modulus is

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h} \quad (7)$$

Values of the shear modulus for some representative materials are given in Table 1. Like Young's modulus, the unit of shear modulus is the ratio of that for force to that for area.

1.2.c Bulk Modulus: Volume Elasticity

Bulk modulus characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object, as ***Bulk modulus*** characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object, as shown in Figure 5. (We assume here that the object is made of a single substance.) such a uniform distribution of forces occurs when an object is immersed in a fluid.

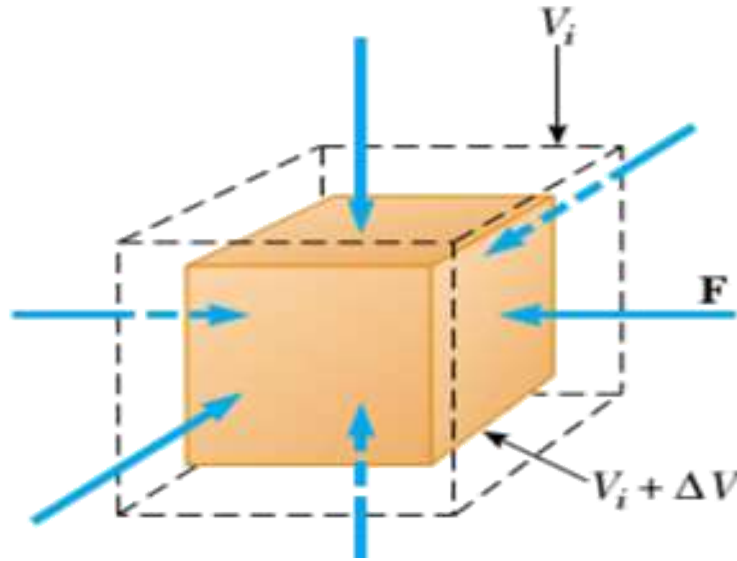


Fig.(5) When a solid is under uniform pressure, it undergoes a change in volume but no change in shape. This cube is compressed on all sides by forces nor-mal to its six faces.

An object subject to this type of deformation undergoes a change in volume but no change in shape. The volume stress is defined as the ratio of the magnitude of the total force $\mathbf{F}$ exerted on a surface to the area A of the surface. The quantity $\mathbf{P} = \mathbf{F}/A$ is called pressure . If the pressure on an object changes by an amount $\Delta\mathbf{P} = \Delta\mathbf{F}/A$, then the object will experience a volume change ΔV .The volume strain is equal to the change in volume ΔV divided by the initial volume V_i . Thus, from Equation 5, we can characterize a volume (“bulk”) compression in terms of the bulk modulus, which is defined as

$$B = \frac{\text{volume stress}}{\text{volume strain}} = - \frac{\Delta F/A}{\Delta V/V_i} = - \frac{\Delta P}{\Delta V/V_i} \quad (8)$$

A negative sign is inserted in this defining equation so that ***B*** is a positive number. *This maneuver is necessary because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa .* Table 1 lists ***bulk moduli*** for some materials. If you look up such values in a different source, you often find that the reciprocal of the ***bulk modulus*** is listed. ***The reciprocal of the bulk modulus is called the compressibility of the material.***

Note from Table 1 that both solids and liquids have a bulk modulus. However, no shear modulus and no Young's modulus are given for liquids because a liquid does not sustain a shearing stress or a tensile stress. If a shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.

2. Harmonic Oscillator

A mass m attached to the end of a spring is a classic example of a harmonic oscillator. By pulling slightly on the mass and then releasing it, the system will be set in sinusoid oscillating motion about the equilibrium position. To the extent that the spring obeys Hooke's law, and that one can neglect friction and the mass of the spring, the amplitude of the oscillation will remain constant; and its

frequency will be independent of its amplitude, determined only by the mass and the stiffness of the spring:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (9)$$

This phenomenon made possible the construction of accurate mechanical clocks and watches that could be carried on ships and people's pockets.

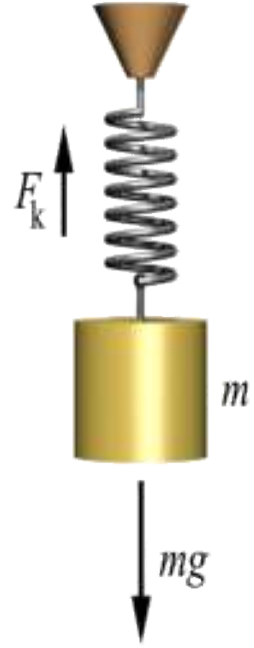


Fig.(6)

2.1 Rotation in Gravity-free Space.

If the mass m were attached to a spring with force constant k and rotating in free space, the spring tension (F_t) would supply the required centripetal force (F_c):

$$F_t = kx \quad (10)$$

$$F_c = m \omega^2 r \quad (11)$$

Since $F_t = F_c$ and $x = r$, then:

$$k = m \omega^2 \quad (12)$$

Given that $\omega = 2\pi f$, this leads to the same frequency equation as above:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (13)$$

3. Universal Gravitation

Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F_g = G \frac{m_1 m_2}{r^2} \hat{r}_{12} \quad (14)$$

where $G=6.673 \times 10^{-11} \text{ N.m}^2 / \text{kg}^2$ is the *universal gravitational constant* . This equation enables us to calculate the force of attraction between masses under a wide variety of circumstances.

It's important point that the magnitude of the force exerted by the Earth on a particle of mass m near the Earth's surface is

$$F_g = G \frac{M_E m}{R_E^2} \quad (15)$$

The magnitude of the gravitational force acting on the object at this position is also $F_g = mg$, where g is the value of the free-fall acceleration at the altitude h .

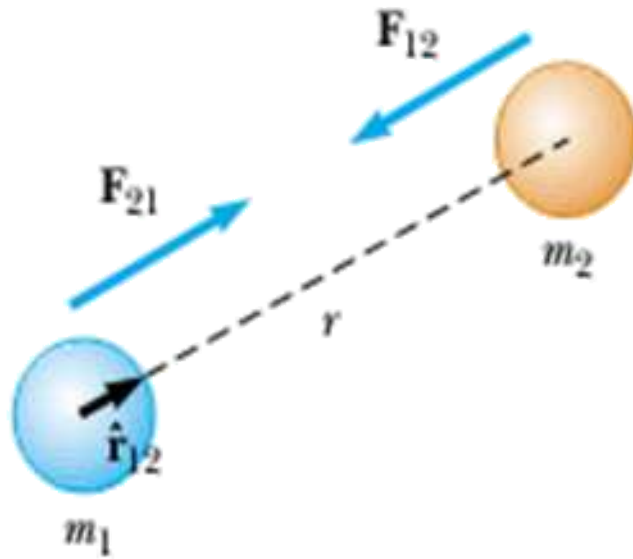


Fig.(7) gravitational force between two particles is attractive. The unit vector $\hat{r}_{12}$ is directed from particle 1 toward particle 2. Note that $F_{21} = -F_{12}$.

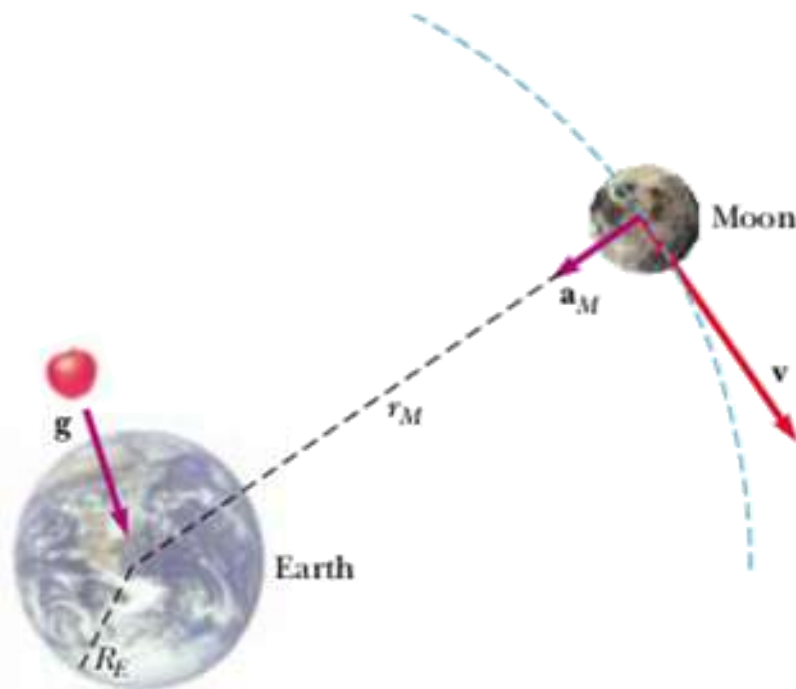


Fig.(8) As it revolves around the Earth, the Moon experiences a centripetal acceleration a_M directed toward the Earth. An object near the Earth's surface, experiences an acceleration g .

Substituting this expression for Fg into the last equation shows that g is

$$g = \frac{G M_E}{r^2} = \frac{G M_E}{(R_E + h)^2} \quad (16)$$

In this expression, M_E is the mass of the Earth and R_E is its radius. Thus, the weight of an object decreases as the object moves away from the Earth's surface.

3.1 Kepler's laws of planetary motion.

The planetary motion state is as the following :

- 1. All planets move in elliptical orbits with the Sun at one focus.*
- 2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.*
- 3. The square of the orbital period of any planet is proportional to the cube of the semi-major axis of the elliptical orbit .*

Therefore, Kepler's third law can be expressed as

$$T^2 = \left(\frac{4\pi^2}{GM_S} \right) a^3 = k a^3 \quad (17)$$

where T is the periodic time around the sun and M_S is the mass of the Sun and a is the semi-major axis, k is constant. For a circular orbit, a can be replaced in Equation 16 by the radius r . Most planets have nearly circular orbits around the Sun.

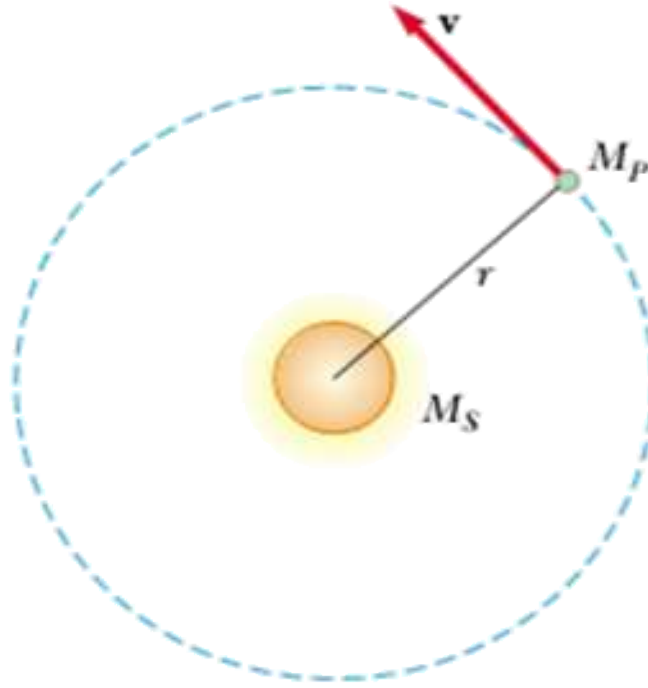


Fig.(9) A planet of mass M_P moving in a circular orbit around the Sun. The orbits of all planets except Mercury and Pluto are nearly circular.

The gravitational field at a point in space is defined as the gravitational force experienced by any test particle located at that point divided by the mass m of the test particle :

$$\mathbf{g} \equiv \frac{\mathbf{F}_g}{m} \quad (18)$$

The gravitational force is conservative, and therefore a potential energy function can be defined for a system of two objects interacting gravitationally. **The gravitational potential energy** associated with two particles separated by a distance r_i is

$$U = - \frac{G m_1 m_2}{r} \quad (19)$$

where U is taken to be zero as $r \rightarrow \infty$. The total potential energy for a system of particles is the sum of energies for all pairs of particles, with each pair represented by a term of the form given by Equation 19. If an isolated system consists of an object of mass m moving with a speed v in the vicinity of a massive object of mass M , the total energy E of the system is the sum of the kinetic and potential energies:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \quad (20)$$

The total energy is a constant of the motion. If the object moves in an elliptical orbit of semi-major axis a around the massive object and if $M \gg m$, the total energy of the system is

$$E = - \frac{GMm}{2a} \quad (21)$$

For a circular orbit, this same equation applies with $a = r$. The total energy is negative for any bound system. The escape speed for an object projected from the surface of a planet of mass M and radius R is

$$v_{esc} = \sqrt{\frac{2GM}{R}} \quad (22)$$

4. Fluid Mechanics

Matter is normally classified as being in one of three states: solid, liquid, or gas. we know that a solid has a definite volume and shape, and a liquid has a definite volume but no definite shape, an unconfined gas has neither a definite volume nor a definite shape. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, a liquid, or a gas.

4.1 Pressure.

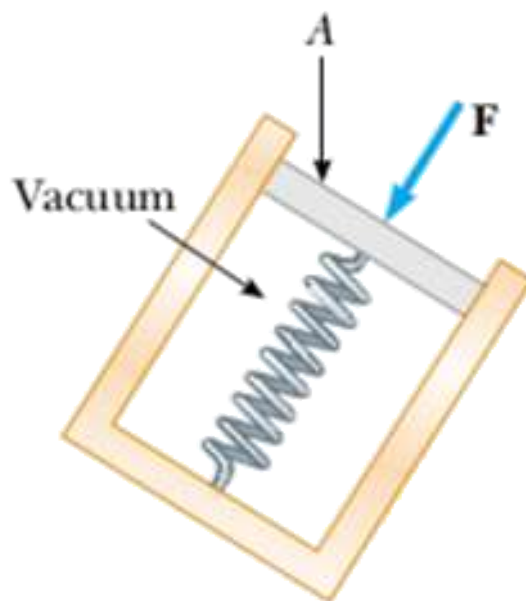


Fig.(7) A simple device for measuring the pressure exerted by a fluid.

If F is the magnitude of the force exerted on the piston, as shown in Fig.(7) and A is the surface area of the piston, then the pressure P of the fluid at the level to which the device has been submerged is defined as the ratio F/A :

$$P \equiv \frac{F}{A}$$

In the SI system, pressure has units of newton's per square meter (N/m^2) , and $1 \text{ N/m}^2 = 1 \text{ pascal(Pa)}$.

4.1.a Variation of Pressure with Depth

The study of the pressure exerted on a fluid, its density is ρ at rest inside a vessel should be proceeded. Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area A extending from depth d to depth $d + h$, as shown in Fig.(8).

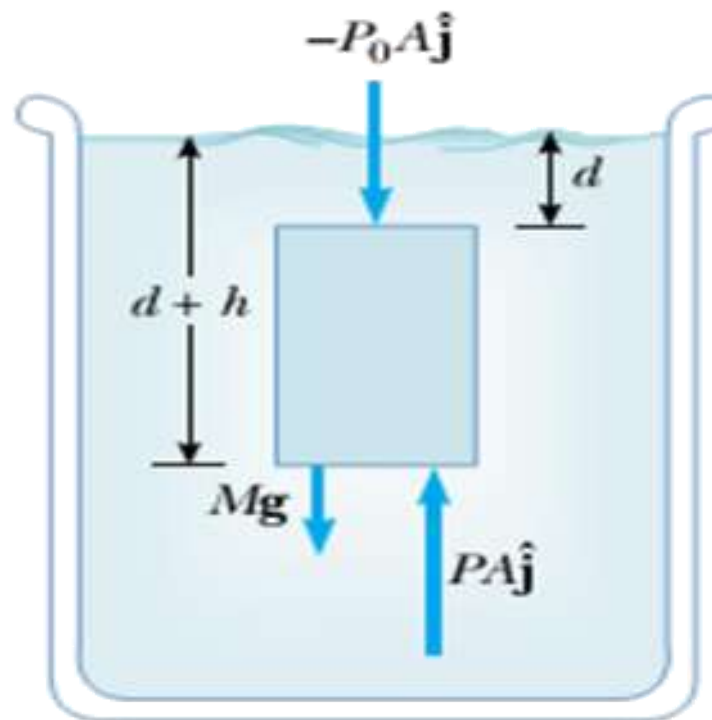


Fig.(8)

The liquid external to our sample exerts forces at all points on the surface of the sample, perpendicular to the surface. The pressure exerted by the liquid on the bottom face of the sample is P , and the pressure on the top face is P_o . The pressure P in a fluid at rest varies with depth h in the fluid according to the expression :

$$P = P_o + \rho gh \quad (23)$$

That is, the pressure P at a depth h below a point in the liquid at which the pressure is P_o is greater by an amount ρgh .

If the liquid is open to the atmosphere and P_o is the pressure at the surface of the liquid, then P_o is atmospheric pressure. Equation 23 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

4.2 Pascal's law states

that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container. An important application of *Pascal's law* is the hydraulic press illustrated in Figure 9. A force of magnitude F_1 is applied to a small piston of surface area A_1 .

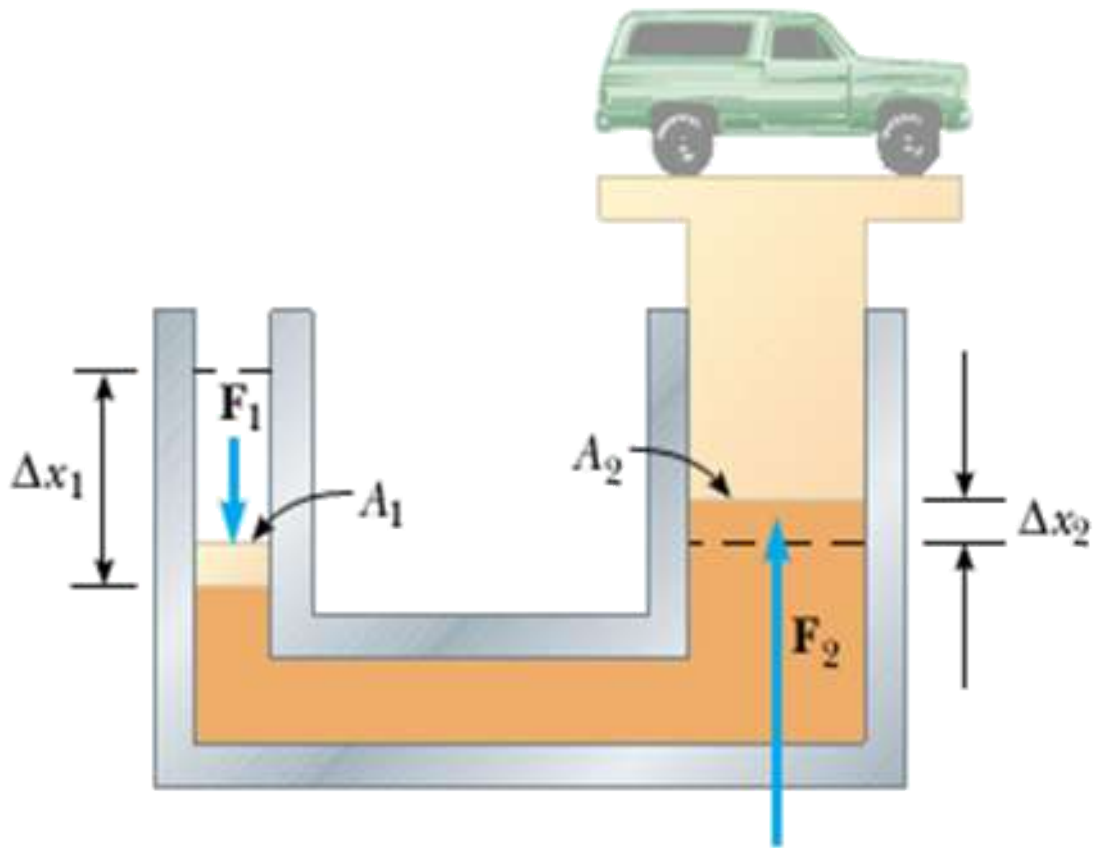
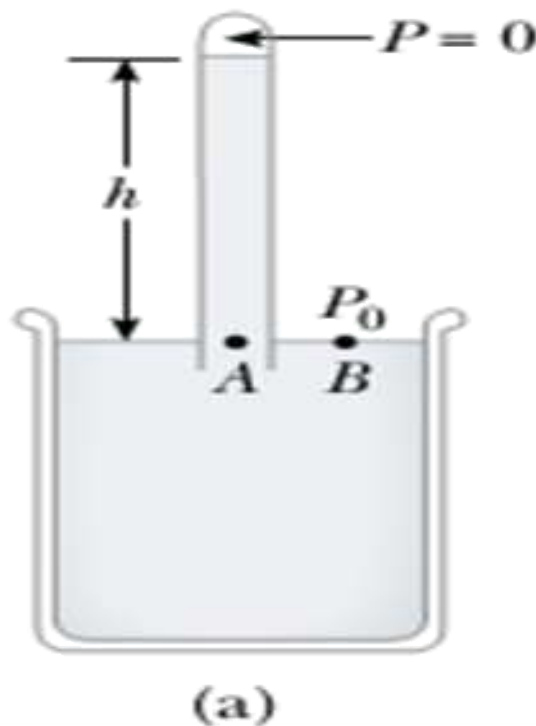


Fig.(9) Diagram of a hydraulic press. Because the increase in pressure is the same on the two sides, a small force F_1 at the left produces a much greater force F_2 at the right.

The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2 . Because the pressure must be the same on both sides, $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is greater than the force F_1 by a factor A_2/A_1 . By designing a hydraulic press with appropriate areas A_1 and A_2 , a large output force can be applied by means of a small input force.

4.3 Pressure Measurements

One instrument used to measure atmospheric pressure is *the common barometer*, invented by Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig. 10). The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. In Figure 14.6a, the pressure at point A, due to the column of mercury, must equal the pressure at point B, due to the atmosphere. Therefore, it follows that $P_o = \rho_{Hg} \cdot g \cdot h$, where ρ_{Hg} is the density of the mercury and h is the height of the mercury column.



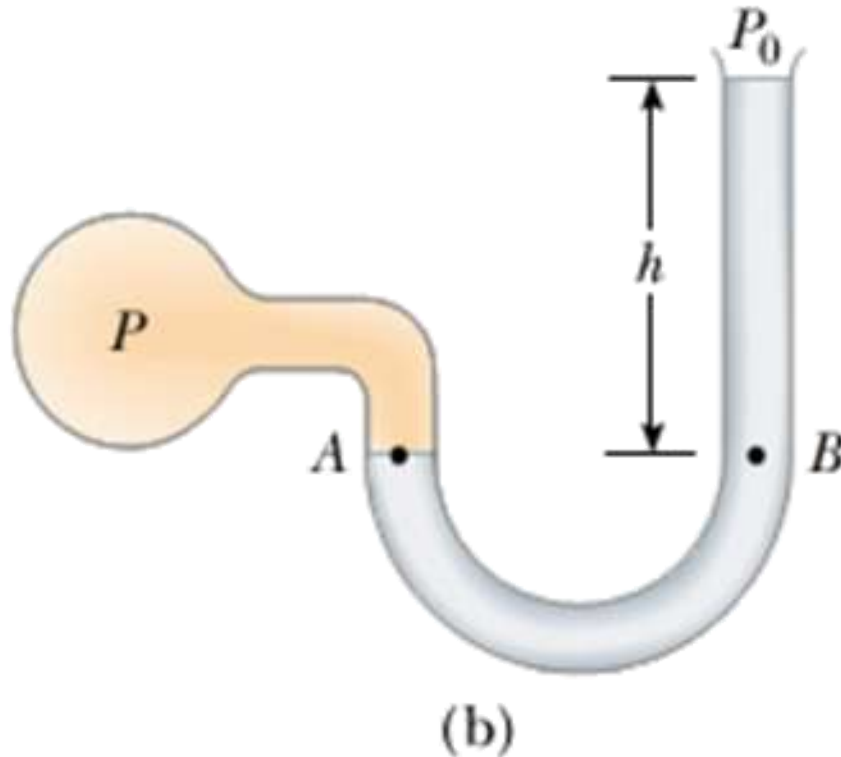


Fig.(10) Two devices for measuring pressure: (a) a mercury barometer and (b) an open-tube manometer

As atmospheric pressure varies, the height of the mercury column varies, so the height can be calibrated to measure atmospheric pressure. Let us determine the height of a mercury column for one atmosphere of pressure, $P_o = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$:

$$P_o = \rho_{Hg} \cdot g \cdot h \rightarrow$$

$$h = \frac{P_o}{\rho_{Hg} g} = \frac{1.013 \times 10^5 \text{ Pa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 0.76 \text{ m}$$

Hence, one atmosphere of pressure is defined to be the pressure equivalent of a column of mercury that is exactly **0.76 m** in height at 0°C.

4.4 Buoyant Forces and Archimedes's Principle.

When an object is partially or fully submerged in a fluid, the fluid exerts on the object an upward force called the buoyant force . According to Archimedes's principle, the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object:

$$B = \rho_{fluid} g \cdot V \quad (25)$$

You can understand various aspects of a fluid's dynamics by assuming that the fluid is non viscous and incompressible, and that the fluid's motion is a steady flow with no rotation Two important concepts regarding ideal fluid flow through a pipe of non- uniform size are as follows:

1. The flow rate (volume flux) through the pipe is constant; this is equivalent to stating that the product of the cross-sectional area A and the speed v at any point is a constant. This result is expressed in *the equation of continuity for fluids* :

$$A_1 \cdot v_1 = A_2 \cdot v_2 = \text{constant} \quad (26)$$

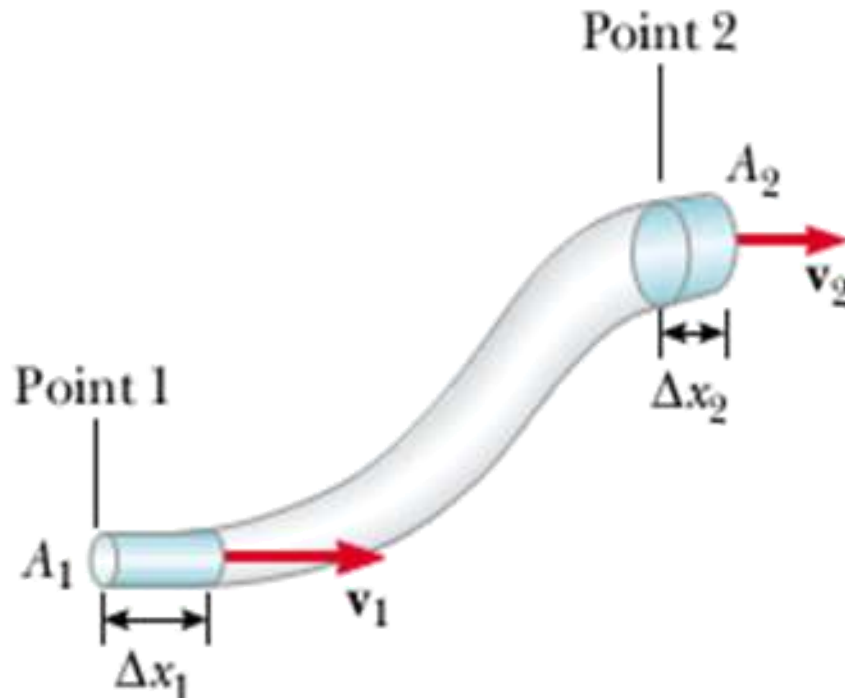


Fig.(11) A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval Δt must equal the volume flowing through area A_2 in the same time interval.

(The equation of continuity for fluids.)

2. The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This result is summarized in *Bernoulli's equation* as :

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant} \quad (27)$$

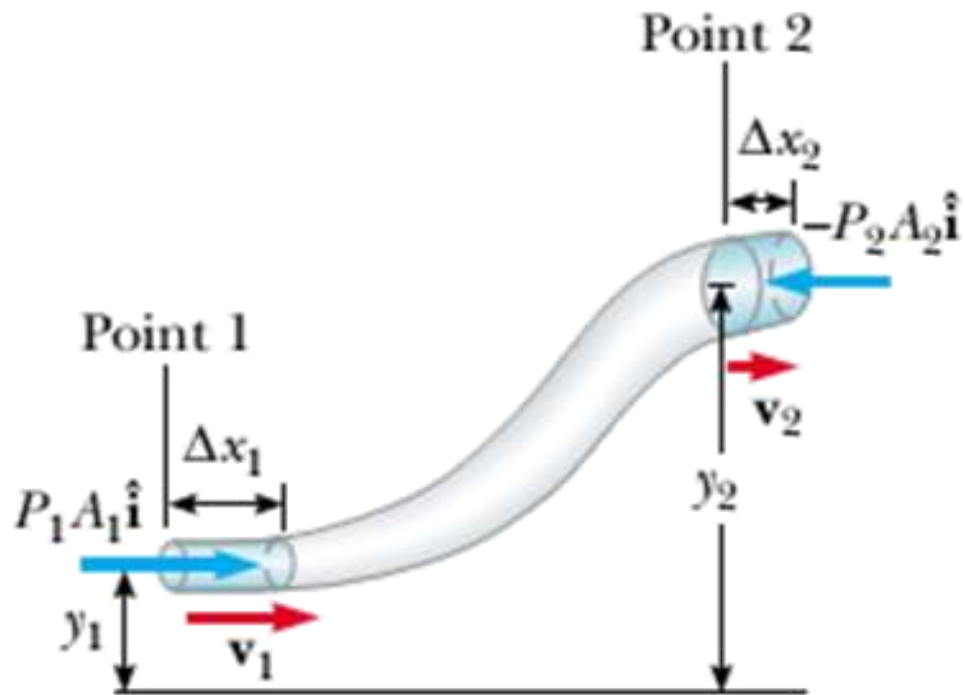


Fig.(12) A fluid in laminar flow through a constricted pipe. The volume of the shaded portion on the left is equal to the volume of the shaded portion on the right.

(Bernoulli's Equation)